



# Classic Word Representations & Vector Space Basics

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## Announcement

- Assignment 1 is due tonight 11:59pm!
- Assignment 2 is released (due 09/25 11:59pm)
- No instructor office hour today

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## Overview of Course Contents

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- Week 1: Logistics & Overview
- Week 2: N-gram Language Models
- **Week 3: Word Senses, Semantics & Classic Word Representations**
- Week 4: Word Embeddings
- Week 5: Sequence Modeling and Transformers
- Week 6-7: Language Modeling with Transformers (Pretraining + Fine-tuning)
- Week 8: Large Language Models (LLMs) & In-context Learning
- Week 9-10: Knowledge in LLMs and Retrieval-Augmented Generation (RAG)
- Week 11: LLM Alignment
- Week 12: Language Agents
- Week 13: Recap + Future of NLP
- Week 15 (after Thanksgiving): Project Presentations



## (Recap) Why Care About Word Semantics?

- Understanding word meanings helps us build better language models!
- Recall the example from N-gram lectures:

[BOS] The cat is on the mat [EOS]

[BOS] I have a cat and a mat [EOS]

[BOS] I like the cat [EOS]

$$p(\text{"cat"}|\text{"the"}) = \frac{2}{3}, \quad p(\text{"mat"}|\text{"the"}) = \frac{1}{3},$$

- Sparsity: many valid bigram counts are zero – count-based measures do not account for word semantics!
- If we know “cat” is semantically similar to “dog”, then  $p(\text{"dog"}|\text{"the"}) \approx p(\text{"cat"}|\text{"the"})$

## (Recap) Word Semantics & Relations in NLP

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- **Synonyms:** words with similar meanings
  - “happy” & “joyful”
- **Antonyms:** words with opposite meanings
  - “hot” & “cold”
- **Hyponyms & hypernyms:** one word is a more specific instance of another
  - “rose” is a hyponym of “flower”
  - “flower” is a hypernym of “rose”
- **Polysemy:** A single word having multiple related meanings
  - “mouse” can mean small rodents or the device that controls a cursor
- **Lemma:** the base or canonical form of a word, from which other forms can be derived
- The study of these aspects of word meanings is called **lexical semantics** in linguistics

## (Recap) Polysemy & Senses

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- **Polysemy:** a single word has multiple related meanings
  - “**Light**”: “This bag is **light**” / “Turn on the **light**” / “She made a **light** comment”
- **Sense:** a particular meaning or interpretation of a word in a given context
- Word relations (e.g., synonyms, antonyms, hypernyms/hyponyms) are defined between word senses!
- **Word sense disambiguation (WSD):** determine which sense of a word is being used in a specific context
  - She went to the **bank** to deposit money
  - She lives by the river **bank**
- WSD can be challenging especially when the context is short/insufficient
  - Is the query “mouse info” looking for a pet or a tool?



## (Recap) Word Similarity

- Most words may not have many perfect synonyms, but usually have lots of similar words
  - “cat” is not a synonym of “dog”, but they are similar in meaning

|        |            |      |
|--------|------------|------|
| vanish | disappear  | 9.8  |
| belief | impression | 5.95 |
| muscle | bone       | 3.65 |
| modest | flexible   | 0.98 |
| hole   | agreement  | 0.3  |

Word similarity (on a scale from 0 to 10)  
manually annotated by humans

- We’ll introduce word embeddings to automatically learn word similarity next week!

## (Recap) Connotation

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- Valence: the pleasantness of the stimulus
  - High: “happy” / “satisfied”; low: “unhappy” / “annoyed”
- Arousal: the intensity of emotion provoked by the stimulus
  - High: “excited”; low: “calm”
- Dominance: the degree of control exerted by the stimulus
  - High: “controlling”; low: “influenced”

|            | Valence | Arousal | Dominance |
|------------|---------|---------|-----------|
| courageous | 8.05    | 5.5     | 7.38      |
| music      | 7.67    | 5.57    | 6.5       |
| heartbreak | 2.45    | 5.65    | 3.58      |
| cub        | 6.71    | 3.95    | 4.24      |

Earliest work on representing words  
with multi-dimensional vectors!



## Agenda

- Introduction to Word Senses & Semantics
- Classic Word Representations
- Vector Space Model Basics

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## WordNet

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- Word semantics is complex (multiple senses, various relations)!
- How did people represent word senses and relations in early NLP developments?
- **WordNet**: A manually curated large lexical database
- Three separate databases: one each for nouns, verbs and adjectives/adverbs
- Each database contains a set of lemmas, each one annotated with a set of senses
- Synset (synonym set): The set of near-synonyms for a sense
- Word relations (hypernym, hyponym, antonym) defined between synsets

# WordNet Relations

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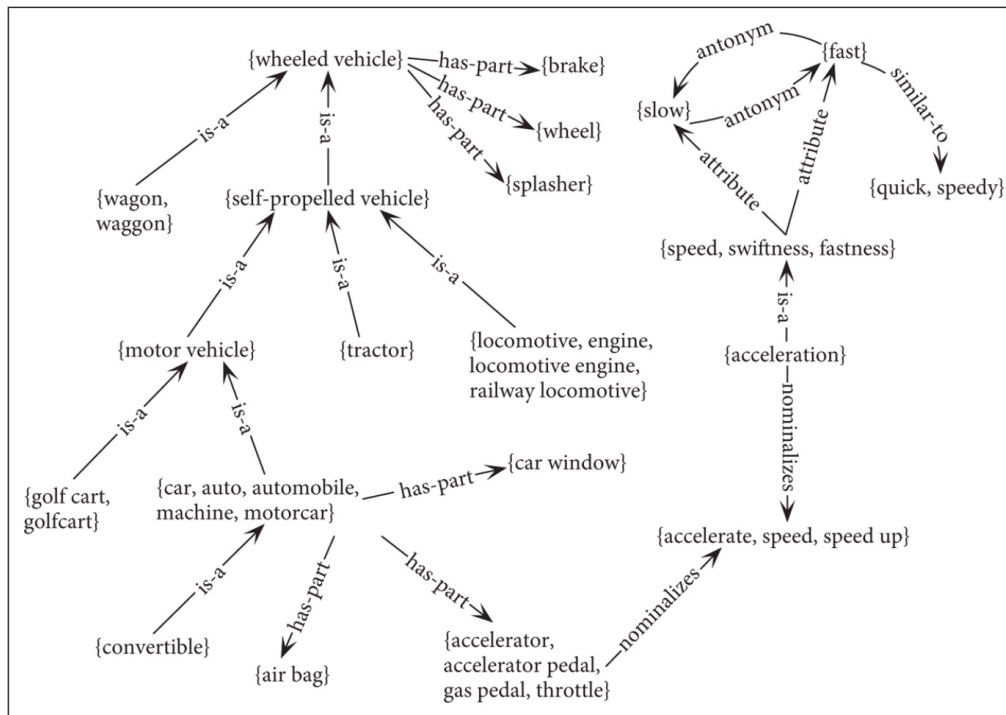

| Relation          | Also Called   | Definition                         | Example   |
|-------------------|---------------|------------------------------------|---|
| Hypernym          | Superordinate | From concepts to superordinates    | <i>breakfast</i> <sup>1</sup> → <i>meal</i> <sup>1</sup>      |
| Hyponym           | Subordinate   | From concepts to subtypes          | <i>meal</i> <sup>1</sup> → <i>lunch</i> <sup>1</sup>          |
| Instance Hypernym | Instance      | From instances to their concepts   | <i>Austen</i> <sup>1</sup> → <i>author</i> <sup>1</sup>       |
| Instance Hyponym  | Has-Instance  | From concepts to their instances   | <i>composer</i> <sup>1</sup> → <i>Bach</i> <sup>1</sup>       |
| Part Meronym      | Has-Part      | From wholes to parts               | <i>table</i> <sup>2</sup> → <i>leg</i> <sup>3</sup>           |
| Part Holonym      | Part-Of       | From parts to wholes               | <i>course</i> <sup>7</sup> → <i>meal</i> <sup>1</sup>         |
| Antonym           |               | Semantic opposition between lemmas | <i>leader</i> <sup>1</sup> ↔ <i>follower</i> <sup>1</sup>     |
| Derivation        |               | Lemmas w/same morphological root   | <i>destruction</i> <sup>1</sup> ↔ <i>destroy</i> <sup>1</sup> |

## Noun relations

| Relation | Definition  | Example   |
|----------|---|---|
| Hypernym | From events to superordinate events                   | <i>fly</i> <sup>9</sup> → <i>travel</i> <sup>5</sup>        |
| Troponym | From events to subordinate event                      | <i>walk</i> <sup>1</sup> → <i>stroll</i> <sup>1</sup>       |
| Entails  | From verbs (events) to the verbs (events) they entail | <i>snore</i> <sup>1</sup> → <i>sleep</i> <sup>1</sup>       |
| Antonym  | Semantic opposition between lemmas                    | <i>increase</i> <sup>1</sup> ↔ <i>decrease</i> <sup>1</sup> |

## Verb relations

# WordNet as a Graph



# WordNet Demo

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| Category  | Unique Strings |
|-----------|----------------|
| Noun      | 117798         |
| Verb      | 11529          |
| Adjective | 22479          |
| Adverb    | 4481           |

Figure source: <https://lm-class.org/lectures/04%20-%20word%20embeddings.pdf>

Word to search for:

Display Options:

Key: "S:" = Show Synset (semantic) relations, "W:" = Show Word (lexical) relations

Display options for sense: (gloss) "an example sentence"

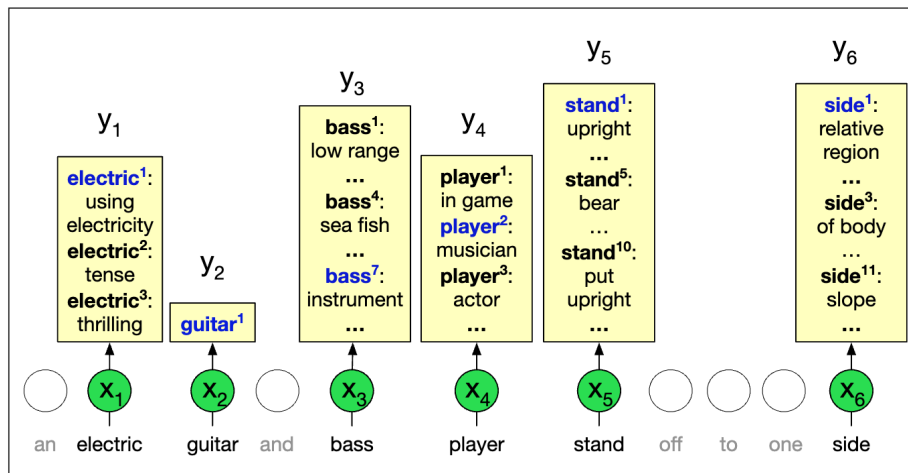
## Noun

- **S: (n) light, visible light, visible radiation** ((physics) electromagnetic radiation that can produce a visual sensation) *"the light was filtered through a soft glass window"*
  - [direct hyponym](#) / [full hyponym](#)
  - [domain category](#)
  - [direct hypernym](#) / [inherited hypernym](#) / [sister term](#)
  - [part holonym](#)
  - [derivationally related form](#)
- **S: (n) light, light source** (any device serving as a source of illumination) *"he stopped the car and turned off the lights"*
- **S: (n) light** (a particular perspective or aspect of a situation) *"although he saw it in a different light, he still did not understand"*
- **S: (n) luminosity, brightness, brightness level, luminance, luminousness, light** (the quality of being luminous; emitting or reflecting light) *"its luminosity is measured relative to that of our sun"*
- **S: (n) light** (an illuminated area) *"he stepped into the light"*
  - [direct hypernym](#) / [inherited hypernym](#) / [sister term](#)
  - [derivationally related form](#)
- **S: (n) light, illumination** (a condition of spiritual awareness; divine illumination) *"follow God's light"*
- **S: (n) light, lightness** (the visual effect of illumination on objects or scenes as created in pictures) *"he could paint the lightest light and the darkest dark"*
- **S: (n) light** (a person regarded very fondly) *"the light of my life"*
- **S: (n) light, lighting** (having abundant light or illumination) *"they played as long as it was light"; "as long as the lighting was good"*
- **S: (n) light** (mental understanding as an enlightening experience) *"he finally saw the light"; "can you shed light on this problem?"*
- **S: (n) sparkle, twinkle, spark, light** (merriment expressed by a brightness or gleam or animation of countenance) *"he had a sparkle in his eye"; "there's a perpetual twinkle in his eyes"*
- **S: (n) light** (public awareness) *"it brought the scandal to light"*
- **S: (n) Inner Light, Light, Light Within, Christ Within** (a divine presence)



# WordNet for Word Sense Disambiguation

- All words WSD task: map all input words (nouns/verbs/adjectives/adverbs) to WordNet senses
- Strong baseline: map to the first sense in WordNet (most frequent)
- Modern approaches: sequence modeling architectures (later lectures!)



## WordNet Limitations

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- Require significant efforts to construct and maintain/update
  - Hard to keep up with rapidly evolving language usage
- Limited coverage of domain-specific terms & low-resource language
  - No coverage of specialized, domain-specific terms (e.g., medical, legal, or technical)
- Only support individual words and their meanings
  - Do not account for idiomatic expressions, phrasal verbs, or collocations

**A more automatic, scalable, and contextualized word semantic learning approach is needed!**

## Agenda

- Introduction to Word Senses & Semantics
- Classic Word Representations
- Vector Space Model Basics

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## Motivation: Representing Texts with Vectors

- Word similarity computation is important for understanding semantics

Word similarity (on a scale from 0 to 10)  
manually annotated by humans

|        |            |      |
|--------|------------|------|
| vanish | disappear  | 9.8  |
| belief | impression | 5.95 |
| muscle | bone       | 3.65 |
| modest | flexible   | 0.98 |
| hole   | agreement  | 0.3  |

Word semantics can be multi-faceted

|            | Valence | Arousal | Dominance |
|------------|---------|---------|-----------|
| courageous | 8.05    | 5.5     | 7.38      |
| music      | 7.67    | 5.57    | 6.5       |
| heartbreak | 2.45    | 5.65    | 3.58      |
| cub        | 6.71    | 3.95    | 4.24      |

- How to represent words numerically? Using multi-dimensional vectors!



## Vector Semantics

- Represent a word as a point in a multi-dimensional semantic space
- A desirable vector semantic space: words with similar meanings are nearby in space



2D visualization of a desirable high-dimensional vector semantic space



## Vector Space Basics

- Vector notation: an N-dimensional vector  $\mathbf{v} = [v_1, v_2, \dots, v_N] \in \mathbb{R}^N$
- Vector dot product/inner product:

$$\text{dot product}(\mathbf{v}, \mathbf{w}) = \mathbf{v} \cdot \mathbf{w} = v_1 w_1 + v_2 w_2 + \dots + v_n w_n = \sum_{i=1}^N v_i w_i$$

- Vector length/norm:

$$|\mathbf{v}| = \sqrt{\mathbf{v} \cdot \mathbf{v}} = \sqrt{\sum_{i=1}^N v_i^2}$$

Other (less commonly-used) vector norms:  
Manhattan norm,  $p$ -norm, infinity norm...

- Cosine similarity between vectors:

$$\cos(\mathbf{v}, \mathbf{w}) = \frac{\mathbf{v} \cdot \mathbf{w}}{|\mathbf{v}| |\mathbf{w}|} = \frac{\sum_{i=1}^N v_i w_i}{\sqrt{\sum_{i=1}^N v_i^2} \sqrt{\sum_{i=1}^N w_i^2}}$$



## Vector Space Basics: Example

- Consider two 4-dimensional vectors  $\mathbf{v} = [1, 0, 1, 0] \in \mathbb{R}^4$     $\mathbf{w} = [0, 1, 1, 0] \in \mathbb{R}^4$
- Vector dot product/inner product:

$$\mathbf{v} \cdot \mathbf{w} = \sum_{i=1}^N v_i w_i = 1$$

- Vector length/norm:

$$|\mathbf{v}| = \sqrt{\sum_{i=1}^N v_i^2} = \sqrt{2} \quad |\mathbf{w}| = \sqrt{\sum_{i=1}^N w_i^2} = \sqrt{2}$$

- Cosine similarity between vectors:

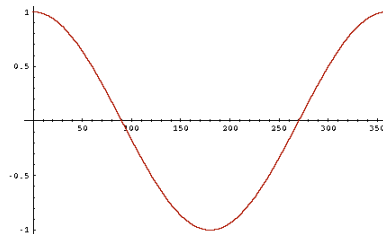
$$\cos(\mathbf{v}, \mathbf{w}) = \frac{\mathbf{v} \cdot \mathbf{w}}{|\mathbf{v}| |\mathbf{w}|} = \frac{1}{2}$$

# Vector Similarity

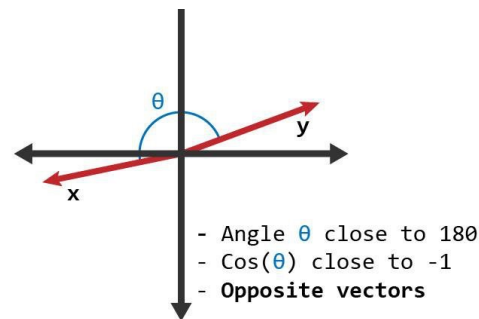
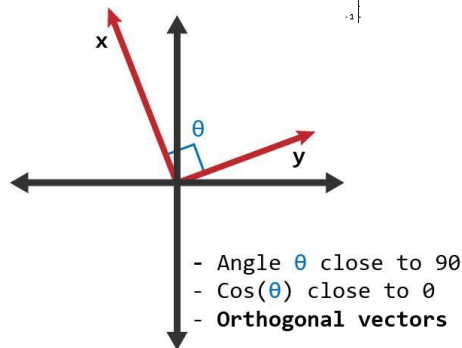
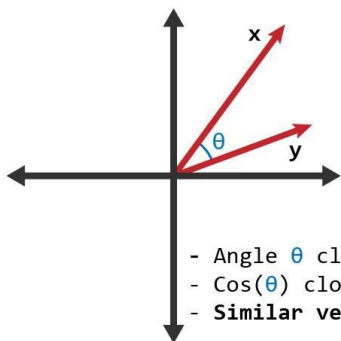
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- Cosine similarity is the most commonly used metric for similarity measurement
  - Symmetric:  $\cos(\mathbf{v}, \mathbf{w}) = \cos(\mathbf{w}, \mathbf{v})$
  - Not influenced by vector length
  - Has a normalized range:  $[-1, 1]$
  - Intuitive geometric interpretation



Cosine function values under different angles





## How to Represent Words as Vectors?

- Given a vocabulary  $\mathcal{V} = \{\text{good, feel, I, sad, cats, have}\}$
- Most straightforward way to represent words as vectors: use their indices
- One-hot vector: only one high value (1) and the remaining values are low (0)
- Each word is identified by a unique dimension

$$\mathbf{v}_{\text{good}} = [1, 0, 0, 0, 0, 0]$$

$$\mathbf{v}_{\text{feel}} = [0, 1, 0, 0, 0, 0]$$

$$\mathbf{v}_{\text{I}} = [0, 0, 1, 0, 0, 0]$$

$$\mathbf{v}_{\text{sad}} = [0, 0, 0, 1, 0, 0]$$

$$\mathbf{v}_{\text{cats}} = [0, 0, 0, 0, 1, 0]$$

$$\mathbf{v}_{\text{have}} = [0, 0, 0, 0, 0, 1]$$



## Represent Sequences by Word Occurrences

- Consider the mini-corpus with three documents

$$d_1 = \text{"I feel good"}$$

$$d_2 = \text{"I feel sad"}$$

$$d_3 = \text{"I have cats"}$$

$$\mathbf{v}_{\text{good}} = [1, 0, 0, 0, 0, 0]$$

$$\mathbf{v}_{\text{feel}} = [0, 1, 0, 0, 0, 0]$$

$$\mathbf{v}_{\text{I}} = [0, 0, 1, 0, 0, 0]$$

$$\mathbf{v}_{\text{sad}} = [0, 0, 0, 1, 0, 0]$$

$$\mathbf{v}_{\text{cats}} = [0, 0, 0, 0, 1, 0]$$

$$\mathbf{v}_{\text{have}} = [0, 0, 0, 0, 0, 1]$$

- Straightforward way of representing documents: look at which words are present

$$\mathbf{v}_{d_1} = [1, 1, 1, 0, 0, 0]$$

$$\mathbf{v}_{d_2} = [0, 1, 1, 1, 0, 0]$$

$$\mathbf{v}_{d_3} = [0, 0, 1, 0, 1, 1]$$

Document vector similarity



$$\cos(\mathbf{v}_{d_1}, \mathbf{v}_{d_2}) = \frac{2}{3}$$

$$\cos(\mathbf{v}_{d_1}, \mathbf{v}_{d_3}) = \frac{1}{3}$$

$$\cos(\mathbf{v}_{d_2}, \mathbf{v}_{d_3}) = \frac{1}{3}$$



## Term-Document Matrix

- With larger text collections, word frequencies in documents entail rich information
- Consider the four plays by Shakespeare and obtain the word frequency statistics
- Look at 4 manually-picked words: “battle” “good” “fool” “wit”

|        | As You Like It | Twelfth Night | Julius Caesar | Henry V |
|--------|----------------|---------------|---------------|---------|
| battle | 1              | 0             | 7             | 13      |
| good   | 114            | 80            | 62            | 89      |
| fool   | 36             | 58            | 1             | 4       |
| wit    | 20             | 15            | 2             | 3       |

There are many more words!

- Document vector representation with word frequencies:

$$\mathbf{v}_{d_1} = [1, 114, 36, 20] \quad \mathbf{v}_{d_2} = [0, 80, 58, 15] \quad \mathbf{v}_{d_3} = [7, 62, 1, 2] \quad \mathbf{v}_{d_4} = [13, 89, 4, 3]$$





## Document Similarity

- Document vector representation with word frequencies:

$$\mathbf{v}_{d_1} = [1, 114, 36, 20] \quad \mathbf{v}_{d_2} = [0, 80, 58, 15] \quad \mathbf{v}_{d_3} = [7, 62, 1, 2] \quad \mathbf{v}_{d_4} = [13, 89, 4, 3]$$

|        | As You Like It | Twelfth Night | Julius Caesar | Henry V |
|--------|----------------|---------------|---------------|---------|
| battle | 1              | 0             | 7             | 13      |
| good   | 114            | 80            | 62            | 89      |
| fool   | 36             | 58            | 1             | 4       |
| wit    | 20             | 15            | 2             | 3       |

- “fool” and “wit” occur much more frequently in  $d_1$  and  $d_2$  than  $d_3$  and  $d_4$
- $d_1$  and  $d_2$  are comedies  $\cos(\mathbf{v}_{d_1}, \mathbf{v}_{d_2}) = 0.95$   $\cos(\mathbf{v}_{d_2}, \mathbf{v}_{d_3}) = 0.81$
- Word frequencies in documents do reflect the semantic similarity between documents!



## Words Represented with Documents

- “Battle”: “the kind of word that occurs in Julius Caesar and Henry V (history plays)”
- “Fool”: “the kind of word that occurs in comedies”

|               | As You Like It | Twelfth Night | Julius Caesar | Henry V |
|---------------|----------------|---------------|---------------|---------|
| <b>battle</b> | 1              | 0             | 7             | 13      |
| <b>good</b>   | 114            | 80            | 62            | 89      |
| <b>fool</b>   | 36             | 58            | 1             | 4       |
| <b>wit</b>    | 20             | 15            | 2             | 3       |

- Represent words using their co-occurrence counts with documents:

$$\mathbf{v}_{\text{battle}} = [1, 0, 7, 13]$$

$$\mathbf{v}_{\text{good}} = [114, 80, 62, 89]$$

$$\mathbf{v}_{\text{fool}} = [36, 58, 1, 4]$$

$$\mathbf{v}_{\text{wit}} = [20, 15, 2, 3]$$



## Words Represented with Documents

|               | As You Like It | Twelfth Night | Julius Caesar | Henry V |
|---------------|----------------|---------------|---------------|---------|
| <b>battle</b> | 1              | 0             | 7             | 13      |
| <b>good</b>   | 114            | 80            | 62            | 89      |
| <b>fool</b>   | 36             | 58            | 1             | 4       |
| <b>wit</b>    | 20             | 15            | 2             | 3       |

$$\mathbf{v}_{\text{battle}} = [1, 0, 7, 13]$$

$$\mathbf{v}_{\text{good}} = [114, 80, 62, 89]$$

$$\mathbf{v}_{\text{fool}} = [36, 58, 1, 4]$$

$$\mathbf{v}_{\text{wit}} = [20, 15, 2, 3]$$



$$\cos(\mathbf{v}_{\text{fool}}, \mathbf{v}_{\text{wit}}) = 0.93$$

$$\cos(\mathbf{v}_{\text{fool}}, \mathbf{v}_{\text{battle}}) = 0.09$$

Previously:

$$\mathbf{v}_{\text{battle}} = [1, 0, 0, 0]$$

$$\mathbf{v}_{\text{good}} = [0, 1, 0, 0]$$

$$\mathbf{v}_{\text{fool}} = [0, 0, 1, 0]$$

$$\mathbf{v}_{\text{wit}} = [0, 0, 0, 1]$$



$$\cos(\mathbf{v}_{\text{fool}}, \mathbf{v}_{\text{wit}}) = 0$$

$$\cos(\mathbf{v}_{\text{fool}}, \mathbf{v}_{\text{battle}}) = 0$$

Document co-occurrence statistics provide coarse-grained contexts

## Fine-Grained Contexts: Word-Word Matrix

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Instead of using documents as contexts for words, we can also use words as contexts

| 4 words to the left               | center word        | 4 words to the right              |
|-----------------------------------|--------------------|-----------------------------------|
| is traditionally followed by      | <b>cherry</b>      | pie, a traditional dessert        |
| often mixed, such as              | <b>strawberry</b>  | rhubarb pie. Apple pie            |
| computer peripherals and personal | <b>digital</b>     | assistants. These devices usually |
| a computer. This includes         | <b>information</b> | available on the internet         |



## Fine-Grained Contexts: Word-Word Matrix

Count how many times words occur in a  $\pm 4$  word window around the center word

context word

center word

|             | aardvark | ... | computer | data | result | pie | sugar | ... |
|-------------|----------|-----|----------|------|--------|-----|-------|-----|
| cherry      | 0        | ... | 2        | 8    | 9      | 442 | 25    | ... |
| strawberry  | 0        | ... | 0        | 0    | 1      | 60  | 19    | ... |
| digital     | 0        | ... | 1670     | 1683 | 85     | 5   | 4     | ... |
| information | 0        | ... | 3325     | 3982 | 378    | 5   | 13    | ... |

Counts derived from the Wikipedia corpus



## Word Similarity Based on Word Co-occurrence

- Word-word matrix with  $\pm 4$  word window

|             | aardvark | ... | computer | data | result | pie | sugar | ... |
|-------------|----------|-----|----------|------|--------|-----|-------|-----|
| cherry      | 0        | ... | 2        | 8    | 9      | 442 | 25    | ... |
| strawberry  | 0        | ... | 0        | 0    | 1      | 60  | 19    | ... |
| digital     | 0        | ... | 1670     | 1683 | 85     | 5   | 4     | ... |
| information | 0        | ... | 3325     | 3982 | 378    | 5   | 13    | ... |

- “digital” and “information” both co-occur with “computer” and “data” frequently
- “cherry” and “strawberry” both co-occur with “pie” and “sugar” frequently
- Word co-occurrence statistics reflect word semantic similarity!
- Issues? Sparsity!



## Is Raw Frequency A Good Representation?

- On the one hand, high frequency can imply semantic similarity
- On the other hand, there are words with universally high frequencies

|               | As You Like It | Twelfth Night | Julius Caesar | Henry V |
|---------------|----------------|---------------|---------------|---------|
| <b>battle</b> | 1              | 0             | 7             | 13      |
| <b>good</b>   | 114            | 80            | 62            | 89      |
| <b>fool</b>   | 36             | 58            | 1             | 4       |
| <b>wit</b>    | 20             | 15            | 2             | 3       |

- Can we reweight the raw frequencies so that distinctively high frequency terms are highlighted?

## Term Frequency (TF)

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- A word appearing 100 times in a document doesn't make it 100 times more likely to be relevant to the meaning of the document
- Instead of using the raw counts, we squash the counts with log scale

$$\text{TF}(w, d) = \begin{cases} 1 + \log_{10} \text{count}(w, d) & \text{count}(w, d) > 0 \\ 0 & \text{otherwise} \end{cases}$$





## Document Frequency (DF)

- Motivation: Give a higher weight to words that occur only in a few documents
  - Terms that are limited to a few documents are more discriminative
  - Terms that occur frequently across the entire collection aren't as helpful
- Document frequency (DF): count how many documents a word occurs in

$$DF(w) = \sum_{i=1}^N \mathbb{1}(w \in d_i) \longrightarrow \begin{array}{l} \text{Evaluates to 1 if } w \text{ occurs in } d_i \\ \text{otherwise evaluates to 0} \end{array}$$

- DF is NOT defined to be the total count of a word across all documents (collection frequency)!

|        | Collection Frequency | Document Frequency |
|--------|----------------------|--------------------|
| Romeo  | 113                  | 1                  |
| action | 113                  | 31                 |



## Inverse Document Frequency (IDF)

- We want to emphasize discriminative words (with low DF)
- Inverse document frequency (IDF): total number of documents (N) divided by DF, in log scale

$$\text{IDF}(w) = \log_{10} \left( \frac{N}{\text{DF}(w)} \right)$$

| Word     | df | idf   |
|----------|----|-------|
| Romeo    | 1  | 1.57  |
| salad    | 2  | 1.27  |
| Falstaff | 4  | 0.967 |
| forest   | 12 | 0.489 |
| battle   | 21 | 0.246 |
| wit      | 34 | 0.037 |
| fool     | 36 | 0.012 |
| good     | 37 | 0     |
| sweet    | 37 | 0     |

DF & IDF statistics in the  
Shakespeare corpus

# TF-IDF Weighting

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The TF-IDF weighted value characterizes the “salience” of a term in a document

$$\text{TF-IDF}(w, d) = \text{TF}(w, d) \times \text{IDF}(w)$$

TF-IDF weighted

|               | As You Like It | Twelfth Night | Julius Caesar | Henry V |
|---------------|----------------|---------------|---------------|---------|
| <b>battle</b> | 0.246          | 0             | 0.454         | 0.520   |
| <b>good</b>   | 0              | 0             | 0             | 0       |
| <b>fool</b>   | 0.030          | 0.033         | 0.0012        | 0.0019  |
| <b>wit</b>    | 0.085          | 0.081         | 0.048         | 0.054   |

$$\cos(\mathbf{v}_{d_2}, \mathbf{v}_{d_3}) = 0.10 \quad \cos(\mathbf{v}_{d_3}, \mathbf{v}_{d_4}) = 0.99$$

Raw counts

|               | As You Like It | Twelfth Night | Julius Caesar | Henry V |
|---------------|----------------|---------------|---------------|---------|
| <b>battle</b> | 1              | 0             | 7             | 13      |
| <b>good</b>   | 114            | 80            | 62            | 89      |
| <b>fool</b>   | 36             | 58            | 1             | 4       |
| <b>wit</b>    | 20             | 15            | 2             | 3       |

$$\cos(\mathbf{v}_{d_2}, \mathbf{v}_{d_3}) = 0.81 \quad \cos(\mathbf{v}_{d_3}, \mathbf{v}_{d_4}) = 0.99$$

## How to Define Documents?

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- The concrete definition of documents is usually open to different design choices
  - Wikipedia article/page
  - Shakespeare play
  - Book chapter/section
  - Paragraph/sentence
  - ...
- Larger documents provide broader context; smaller ones provide focused insights
- Depends on the analysis need: interested in global trends across documents (e.g., news articles) vs. more local patterns (e.g., specific sections of a legal document)?



**Thank You!**

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