Word2Vec

Slido: https://app.sli.do/event/8boTz6exoJaBx4p6JBkU4x

Yu Meng
University of Virginia
yumeng5@virginia.edu

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Overview of Course Contents

- Week 1: Logistics & Overview
- Week 2: N-gram Language Models
- Week 3: Word Senses, Semantics & Classic Word Representations
- Week 4: Word Embeddings
- Week 5: Sequence Modeling & Recurrent Neural Networks (RNNs)
- Week 6: Language Modeling with Transformers
- Week 9: Large Language Models (LLMs) & In-context Learning
- Week 10: Knowledge in LLMs and Retrieval-Augmented Generation (RAG)
- Week 11: LLM Alignment
- Week 12: Reinforcement Learning for LLM Post-Training
- Week 13: LLM Agents + Course Summary
- Week 15 (after Thanksgiving): Project Presentations

Reminder

- Assignment 2 is due today 11:59pm

(Recap) Vector Space Models

- Vector semantic space: use vector representations to reflect word semantics
- Cosine similarity is the most-commonly used metric for vector similarity
- Word-document & word-word co-occurrence statistics provide valuable semantic information – count-based vector representations work decently well
- Raw counts are not good representations (e.g., biased to universally frequent terms)
- TF-IDF highlights the important words in a document relative to other documents
- PMI measures the strength of association between two words based on probabilistic (independence) assumptions

(Recap) Count-based Vector Limitations

- Count-based vectors are **sparse** (lots of zeros)
  - Zero values in the vectors do not carry any semantics
- Count-based vectors are **long** (many dimensions)
  - Vector dimension = vocabulary size (usually > 10K)
  - "Curse of dimensionality": metrics (e.g. cosine) become less meaningful in high dimensions

	aardvark	...	computer	data	result	pie	sugar	...
cherry	0	...	2	8	9	442	25	...
strawberry	0	...	0	0	1	60	19	...
digital	0	...	1670	1683	85	5	4	...
information	0	...	3325	3982	378	5	13	...

Many more words!

(Recap) Dense Vectors

- More efficient & effective vector representations?
- **Dense** vectors!
  - Most/all dimensions in the vectors are non-zero
  - Usually floating-point numbers; each dimension could be either positive or negative
  - Dimension much smaller than sparse vectors (i.e., << 10K)
- Also called "**distributed** representations"
  - The information is **distributed** across multiple units/dimensions
  - Each unit/dimension participates in representing multiple pieces of information
  - Analogous to human brains: the brain stores and processes information in a distributed manner: instead of having a single neuron/region represent a concept, information is represented across a network of neurons

(Recap) Dense Vector Example

- One dimension might (partly) contribute to distinguishing animals (“cat” “dog”) from vehicles (“car” “truck”)
- One dimension might (partly) capture some aspect of size
- Another might (partly) represent formality or emotional tone
- …
- Each of these dimensions is not exclusively responsible for any single concept, but together, they combine to form a rich and nuanced representation of words!

$$\boldsymbol{v}_{\text{good}} = [-1.34, 2.58, 0.37, 4.32, -3.21, \ldots]$$
$$\boldsymbol{v}_{\text{nice}} = [-0.58, 1.97, 0.20, 3.13, -2.58, \ldots]$$

Only showing two decimal places
(typically they are floating point numbers!)

(Recap) Dense Vectors Pros & Cons

- **(+) Compactness**: Represent a large number of concepts using fewer resources (richer semantic information per dimension); easier to use as features to neural networks
- **(+) Robustness**: Information is spread across many dimensions => more robust to the randomness/noise in individual units
- **(+) Scalability & Generalization**: Efficiently handle large-scale data and generalize to various applications
- **(-) Lack of Interpretability**: (Unlike sparse vectors) difficult to assign a clear meaning to individual dimensions, making model interpretation challenging

(Recap) Distributional Hypothesis

- Words that occur in similar contexts tend to have similar meanings
- A word’s meaning is largely defined by the company it keeps (its context)
- Example: suppose we don’t know the meaning of “Ong choy” but see the following:
  - Ong choy is delicious **sautéed with garlic**
  - Ong choy is superb **over rice**
  - … ong choy **leaves** with **salty** sauces
- And we’ve seen the following contexts:
  - … spinach **sautéed with garlic over rice**
  - … chard stems and **leaves** are **delicious**
  - … collard greens and other **salty** leafy greens
- Ong choy = water spinach!

Example source: https://web.stanford.edu/~jurafsky/slp3/slides/vectorsemantics2024.pdf

(Recap) Word Embeddings: General Idea

- Learn dense vector representations of words based on distributional hypothesis
- Semantically similar words (based on context similarity) will have similar vector representations
- **Embedding**: a mapping that takes elements from one space and represents them in a different space

$$
\begin{aligned}
\boldsymbol{v}_{\text{to}} &= [1, 0, 0, 0, 0, 0, \ldots] \\
\boldsymbol{v}_{\text{by}} &= [0, 1, 0, 0, 0, 0, \ldots] \\
\boldsymbol{v}_{\text{that}} &= [0, 0, 1, 0, 0, 0, \ldots] \\
\boldsymbol{v}_{\text{good}} &= [0, 0, 0, 1, 0, 0, \ldots] \\
\boldsymbol{v}_{\text{nice}} &= [0, 0, 0, 0, 1, 0, \ldots] \\
\boldsymbol{v}_{\text{bad}} &= [0, 0, 0, 0, 0, 1, \ldots]
\end{aligned}
$$

2D visualization of a word embedding space

Figure source: https://web.stanford.edu/~jurafsky/slp3/6.pdf

(Recap) Learning Word Embeddings

- Assume a large text collection (e.g., Wikipedia)
- Hope to learn similar word embeddings for words occurring in similar contexts
- Construct a prediction task: use a center word's embedding to predict its contexts!
- Intuition: If two words have similar embeddings, they will predict similar contexts, thus being semantically similar!

Predicted contexts

$\boldsymbol{v}_{\text{ong choy}}$ → sautéed, garlic, rice, salty, leaves, …

Predicted contexts

$\boldsymbol{v}_{\text{spinach}}$ → sautéed, garlic, rice, salty, leaves, …

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(Recap) Word Embedding Is Self-Supervised Learning

- **Self-supervised learning**: a model learns to predict parts of its input from other parts of the same input

Input: *Ong choy is superb over rice*

Prediction task: Ong choy → is, superb, over, rice

- Self-supervised learning vs. supervised learning:
  - Self-supervised learning: **no human-labeled data** – the model learns from unlabeled data by generating supervision through the structure of the data itself
  - Supervised learning: **use human-labeled data** – the model learns from human annotated input-label pairs

(Recap) Word Embedding as Input Features

Word embeddings are commonly used as input features to language models

hidden states

$\boldsymbol{h}^{(t)} = \sigma\left(\boldsymbol{W}_h \boldsymbol{h}^{(t-1)} + \boldsymbol{W}_e \boldsymbol{e}^{(t)} + \boldsymbol{b}_1\right)$

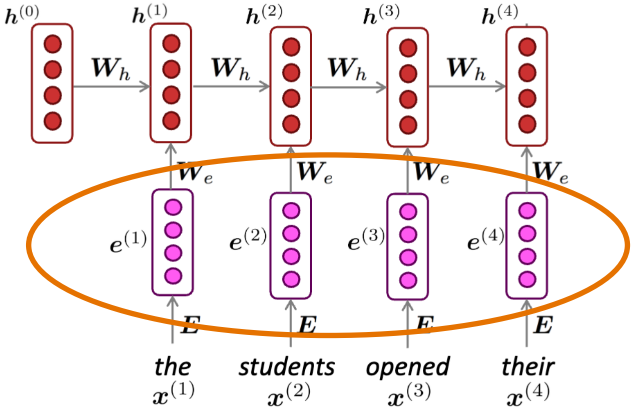
$\boldsymbol{h}^{(0)}$ is the initial hidden state

word embeddings

$\boldsymbol{e}^{(t)} = \boldsymbol{E}\boldsymbol{x}^{(t)}$

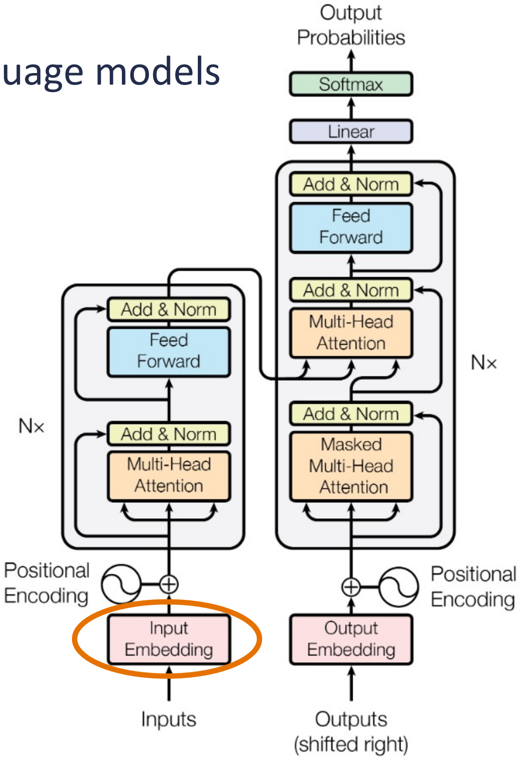
words / one-hot vectors

$\boldsymbol{x}^{(t)} \in \mathbb{R}^{|V|}$



***Note**: this input sequence could be much longer now!*

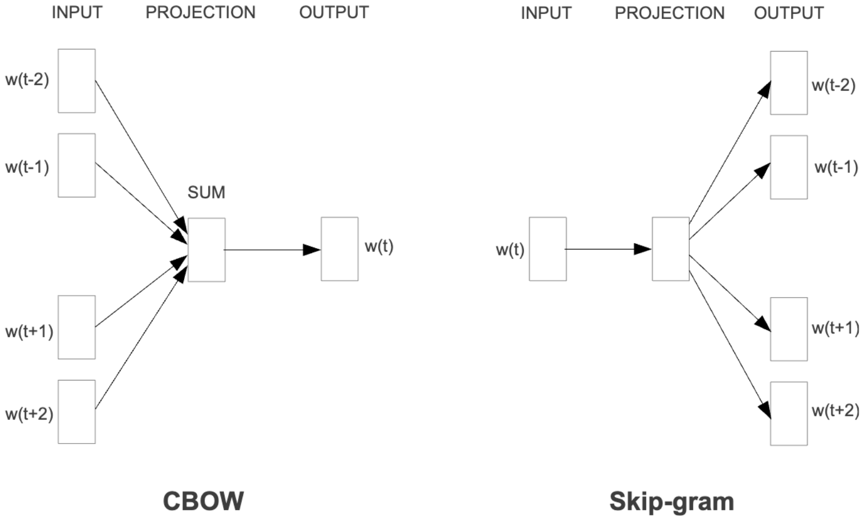
RNN Language Model:
https://web.stanford.edu/class/cs224n/slides/cs224n-spr2024-lecture05-rnnlm.pdf



Transformer: https://arxiv.org/pdf/1706.03762

(Recap) Word2Vec Overview

- The earliest & most well-known word embedding learning method (published in 2013)
- Two variants: Skip-gram and CBOW (Continuous Bag-of-Words)
- We will mainly cover Skip-gram in this lecture



Word2Vec paper: https://arxiv.org/pdf/1301.3781

Word2Vec Setting

- Input: a corpus D – the larger, the better!
- Training data: word-context pairs (w, c) where w is a center word, and c is a context word
    - Each word in the corpus can act as center word
    - Context words = neighboring words of the center word in a local context window ($\pm l$ words)
- Parameters to learn: $\boldsymbol{\theta} = \{\boldsymbol{v}_{\boldsymbol{w}}, \boldsymbol{v}_{\boldsymbol{c}}\}$ – each word has two vectors (center word representation & context word representation)
- The center word representations $\boldsymbol{v}_{\boldsymbol{w}}$ are usually used as the final word embeddings
- Number of parameters to store: $d \times |V|$
    - d is the embedding dimension; usually 100-300
    - $|V|$ is the vocabulary size; usually > 10K
    - Sparse vector representations will have $|V|^2$ parameters!

Word2Vec Training Data Example

- Input sentence: “there is a cat on the mat”
- Suppose context window size = 2
- Word-context pairs as training data:
  - (there, is), (there, a)
  - (is, there), (is, a), (is, cat)
  - (a, there), (a, is), (a, cat), (a, on)
  - (cat, is), (cat, a), (cat, on), (cat, the)
  - (on, a), (on, cat), (on, the), (on, mat)
  - (the, cat), (the, on), (the, mat)
  - (mat, on), (mat, the)

there is a cat on the mat
there is a cat on the mat
there is a cat on the mat
there is a cat on the mat
there is a cat on the mat
there is a cat on the mat
there is a cat on the mat

- “Skip-gram”: skipping over some context words to predict the others!
- Training data completely derived from the raw corpus (no human labels!)

Word2Vec Objective (Skip-gram)

- Intuition: predict the contexts words using the center word (semantically similar center words will predict similar contexts words)
- Objective: using the parameters $\boldsymbol{\theta} = \{\boldsymbol{v_w}, \boldsymbol{v_c}\}$ to maximize the probability of predicting the context word c using the center word w

$$\max_{\boldsymbol{\theta}} \prod_{(w,c)\in\mathcal{D}} \boxed{p_{\boldsymbol{\theta}}(c|w)}$$

Probability expressed as a function of the model parameters

- How to parametrize the probability?

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Word2Vec Probability Parametrization

- Word2Vec objective: $\max_{\boldsymbol{\theta}} \prod_{(w,c)\in\mathcal{D}} p_{\boldsymbol{\theta}}(c|w)$
- Assume the log probability (i.e., logit) is proportional to vector dot product

$$\log p_{\boldsymbol{\theta}}(c|w) \propto \boldsymbol{v}_c \cdot \boldsymbol{v}_w$$

- Rationale: a larger vector dot product *can* indicate a higher vector similarity
- Why not use cosine similarity?
  - Cosine similarity is a non-linear function; more complicated to optimize than dot product
  - With advanced optimization techniques, optimizing cosine similarity is more beneficial (Meng et al.)

Spherical Text Embedding: https://arxiv.org/pdf/1911.01196

Word2Vec Parameterized Objective

- Word2Vec objective: $$\max_{\boldsymbol{\theta}} \prod_{(w,c)\in\mathcal{D}} p_{\boldsymbol{\theta}}(c|w)$$
- Assume the log probability (i.e., logit) is proportional to vector dot product
$$\log p_{\boldsymbol{\theta}}(c|w) \propto \boldsymbol{v}_c \cdot \boldsymbol{v}_w$$
- The final probability distribution is given by the softmax function:
$$p_{\boldsymbol{\theta}}(c|w) = \frac{\exp(\boldsymbol{v}_c \cdot \boldsymbol{v}_w)}{\sum_{c'\in|\mathcal{V}|} \exp(\boldsymbol{v}_{c'} \cdot \boldsymbol{v}_w)} \Rightarrow \sum_{c'\in|\mathcal{V}|} p_{\boldsymbol{\theta}}(c'|w) = 1$$
- Word2Vec objective (log-scale):
$$\max_{\boldsymbol{\theta}} \sum_{(w,c)\in\mathcal{D}} \log p_{\boldsymbol{\theta}}(c|w) = \sum_{(w,c)\in\mathcal{D}} \left(\boldsymbol{v}_c \cdot \boldsymbol{v}_w - \log \sum_{c'\in|\mathcal{V}|} \exp(\boldsymbol{v}_{c'} \cdot \boldsymbol{v}_w) \right)$$

Word2Vec Negative Sampling

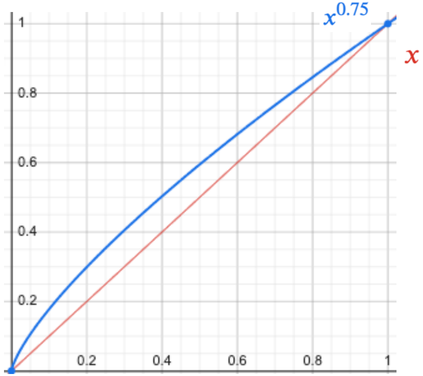
- Challenges with the original objective: Sum over the entire vocabulary – expensive!

$$\max_{\boldsymbol{\theta}} \sum_{(w,c)\in\mathcal{D}} \log p_{\boldsymbol{\theta}}(c|w) = \sum_{(w,c)\in\mathcal{D}} \left(\boldsymbol{v}_c \cdot \boldsymbol{v}_w - \log \boxed{\sum_{c'\in|\mathcal{V}|}} \exp(\boldsymbol{v}_{c'} \cdot \boldsymbol{v}_w) \right)$$

- Randomly sample a few negative terms from the vocabulary to form a negative set N
- How to sample negatives? Based on the (power-smoothed) unigram distribution

$$p_{\text{neg}}(w) \propto \left(\frac{\#(w)}{\sum_{w'\in\mathcal{V}} \#(w')} \right)^{0.75}$$

Rare words get a bit boost in sampling probability



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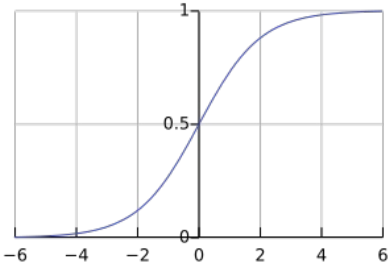
Word2Vec Negative Sampling

- Formulate a binary classification task; predict whether (w, c) is a real context pair:

$$p_{\boldsymbol{\theta}}(\text{True}|c, w) = \sigma(\boldsymbol{v}_c \cdot \boldsymbol{v}_w) = \frac{1}{1 + \exp(-\boldsymbol{v}_c \cdot \boldsymbol{v}_w)}$$



$y = \sigma(x)$

- Maximize the binary classification probability for real context pairs, and minimize for negative (random) pairs

$$\max_{\boldsymbol{\theta}} \underbrace{\log \sigma(\boldsymbol{v}_c \cdot \boldsymbol{v}_w)}_{\text{Real context pair}} - \sum_{c' \in \mathcal{N}} \underbrace{\log \sigma(\boldsymbol{v}_{c'} \cdot \boldsymbol{v}_w)}_{\text{Negative context pair}}$$

Word2Vec Optimization

- How to optimize the following objective?

$$\max_{\boldsymbol{\theta}} \log \sigma(\boldsymbol{v}_c \cdot \boldsymbol{v}_w) - \sum_{c' \in \mathcal{N}} \log \sigma(\boldsymbol{v}_{c'} \cdot \boldsymbol{v}_w)$$

- Stochastic gradient descent (SGD)!
- First, initialize parameters $\boldsymbol{\theta} = \{\boldsymbol{v}_w, \boldsymbol{v}_c\}$ with random d-dimensional vectors
- In each step: update parameters in the direction of the gradient of the objective (weighted by the learning rate)

$$\boldsymbol{\theta}^{(t+1)} \leftarrow \boldsymbol{\theta}^{(t)} - \underbrace{\eta}_{\text{Learning rate}} \nabla_{\boldsymbol{\theta}} \underbrace{\mathcal{L}}_{\text{Loss function}}\Big|_{\boldsymbol{\theta}=\boldsymbol{\theta}^{(t)}}$$

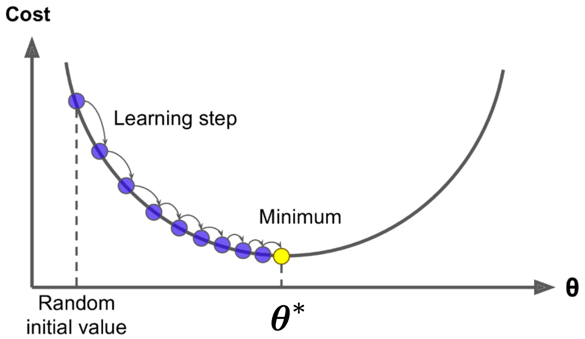


Figure source: https://web.stanford.edu/class/cs224n/slides/cs224n-spr2024-lecture02-wordvecs2.pdf

Word2Vec Hyperparameters

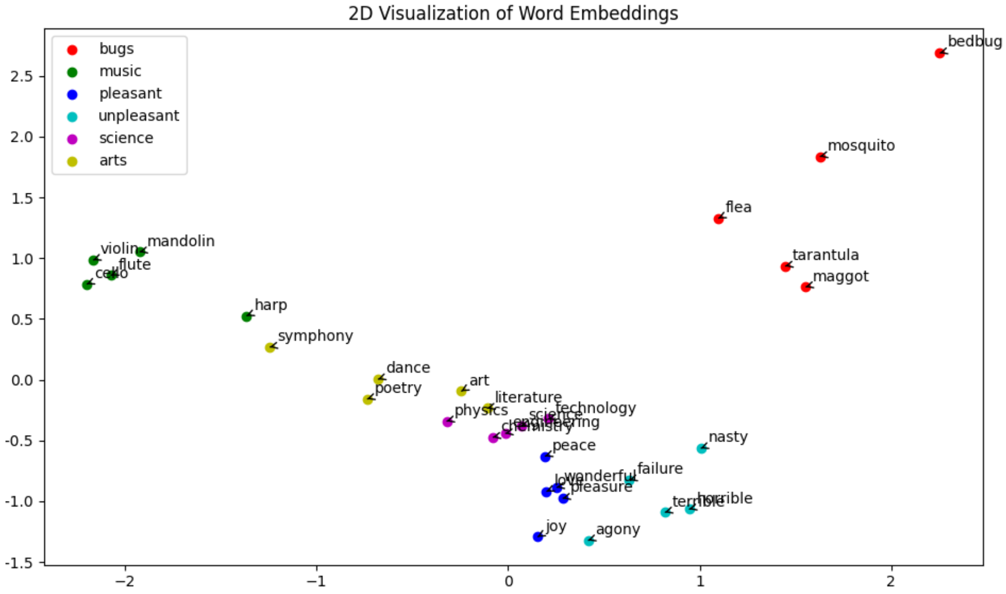
- Word embedding dimension d (usually 100-300)
  - Larger d provides richer vector semantics
  - Extremely large d suffers from inefficiency and curse of dimensionality
- Local context window size l (usually 5-10)
  - Smaller l learns from immediately nearby words – more syntactic information
  - Bigger l learns from longer-ranged contexts – more semantic/topical information
- Number of negative samples k (usually 5-10)
  - Larger k usually makes training more stable but also more costly
- Learning rate η (usually 0.02-0.05)

Agenda

- Sparse vs. Dense Vectors
- Word Embeddings: Overview
- Word2Vec Training
- Word Embedding Properties & Evaluation
- Other Word Embedding Methods
- Word Embedding Limitations & Summary

Word Similarity

- Measure word similarity with cosine similarity between embeddings $\cos(\boldsymbol{v}_{w_1}, \boldsymbol{v}_{w_2})$
- Higher cosine similarity = more semantically close



Word Similarity Evaluation

- An **intrinsic** word embedding evaluation
- Measure how well word vector similarity correlates with human judgments
- Example dataset: WordSim353 (353 word pairs with their similarity scores assessed by humans)

Word 1	Word 2	Human (mean)
tiger	cat	7.35
book	paper	7.46
computer	internet	7.58
plane	car	5.77
professor	doctor	6.62
stock	phone	1.62
stock	CD	1.31
stock	jaguar	0.92

Figure source: https://web.stanford.edu/class/cs224n/slides/cs224n-spr2024-lecture02-wordvecs2.pdf

Correlation Metric

Spearman rank correlation: measure the correlation between two rank variables

Word 1	Word 2	Human (mean)
tiger	cat	7.35
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plane	car	5.77
professor	doctor	6.62
stock	phone	1.62
stock	CD	1.31
stock	jaguar	0.92

Rank by human

$$\begin{bmatrix} 6 \\ 7 \\ 8 \\ 4 \\ 5 \\ 3 \\ 2 \\ 1 \end{bmatrix}$$

$$\mathrm{Cov}(X, Y) = \mathbb{E}[(X - \mathbb{E}[X])(Y - \mathbb{E}[Y])]$$

Covariance

$$r = \frac{\mathrm{Cov}[R[X], R[Y]]}{\sigma_{R[X]}\sigma_{R[Y]}}$$

Standard deviations

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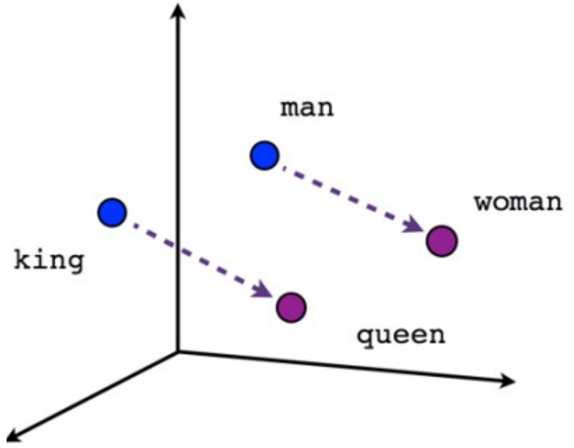
Word Analogy

- Word embeddings reflect intuitive semantic and syntactic analogy
- Example: man : woman :: king : ? $\boldsymbol{v}_{\text{queen}} \approx \boldsymbol{v}_{\text{woman}} - \boldsymbol{v}_{\text{man}} + \boldsymbol{v}_{\text{king}}$
- General case: find the word such that a : b :: c : ?
- Find the word that maximizes the cosine similarity

$$w = \arg\max_{w' \in \mathcal{V}} \cos(\boldsymbol{v}_b - \boldsymbol{v}_a + \boldsymbol{v}_c, \boldsymbol{v}_{w'})$$
$$= \arg\max_{w' \in \mathcal{V}} \frac{(\boldsymbol{v}_b - \boldsymbol{v}_a + \boldsymbol{v}_c) \cdot \boldsymbol{v}_{w'}}{|\boldsymbol{v}_b - \boldsymbol{v}_a + \boldsymbol{v}_c||\boldsymbol{v}_{w'}|}$$



Word Analogy Evaluation

- Word analogy is another **intrinsic** word embedding evaluation
- Encompass various types of word relationships
- Usually use accuracy as the metric

Type of relationship	Word Pair 1		Word Pair 2	
Common capital city	Athens	Greece	Oslo	Norway
All capital cities	Astana	Kazakhstan	Harare	Zimbabwe
Currency	Angola	kwanza	Iran	rial
City-in-state	Chicago	Illinois	Stockton	California
Man-Woman	brother	sister	grandson	granddaughter
Adjective to adverb	apparent	apparently	rapid	rapidly
Opposite	possibly	impossibly	ethical	unethical
Comparative	great	greater	tough	tougher
Superlative	easy	easiest	lucky	luckiest
Present Participle	think	thinking	read	reading
Nationality adjective	Switzerland	Swiss	Cambodia	Cambodian
Past tense	walking	walked	swimming	swam
Plural nouns	mouse	mice	dollar	dollars
Plural verbs	work	works	speak	speaks

Figure source: https://arxiv.org/pdf/1301.3781

Extrinsic Evaluation of Word Embeddings

- Word embeddings can be used as input features to task-specific NLP models
- Example 1: Text classification (topic/sentiment classification)
  - Sentence/document embeddings are obtained by applying sequence modeling architectures on top of word embeddings
  - Classification accuracy is used as the extrinsic metric
- Example 2: Named entity recognition (NER)
  - Find and classify entity names (e.g., person, organization, location) in text
  - Concatenated word embeddings can be used to represent spans of words (entities)
  - Precision/recall/F1 are used as the extrinsic metrics
- Word embedding demo

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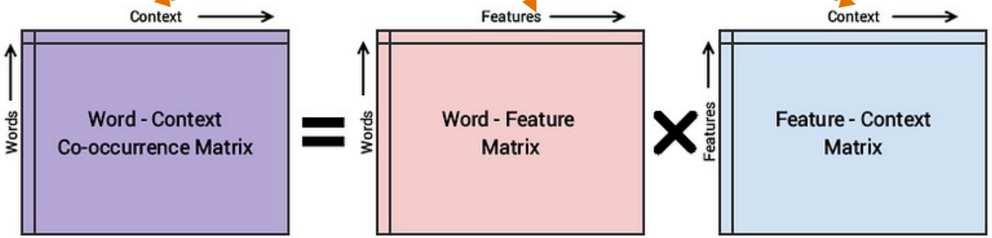
GloVe: Global Vectors for Word Representation

- Core insight: ratios of co-occurrence probabilities can encode meaning components

Probability and Ratio	$k = solid$	$k = gas$	$k = water$	$k = fashion$
$P(k\|ice)$	1.9×10^{-4}	6.6×10^{-5}	3.0×10^{-3}	1.7×10^{-5}
$P(k\|steam)$	2.2×10^{-5}	7.8×10^{-4}	2.2×10^{-3}	1.8×10^{-5}
$P(k\|ice)/P(k\|steam)$	8.9	8.5×10^{-2}	1.36	0.96

- Training objective: $$J = \sum_{i,j=1}^{V} f\left(X_{ij}\right)\left(w_i^T \tilde{w}_j + b_i + \tilde{b}_j - \log X_{ij}\right)^2$$



A (reweighted) matrix factorization problem

GloVe: https://aclanthology.org/D14-1162.pdf

FastText: Incorporating Subword Information

- Motivation: treating each word as a whole ignores the internal structure of words
- Solution: representing words with character N-grams
- Example (assume character trigram):
  - the word “where” will be decomposed into: `<wh, whe, her, ere, re>`
  - The word “her” will be represented as `<her>`
- Each word is represented by the sum of the vectors of its character N-grams
- Use the same training objective as Word2Vec
- Benefit: more robust representations for rare words

FastText: https://arxiv.org/pdf/1607.04606

Word Embedding: Further Reading

- Neural Word Embedding as Implicit Matrix Factorization [Levy & Goldberg, 2014]
- Distributed Representations of Sentences and Documents [Le & Mikolov, 2014]
- Poincaré Embeddings for Learning Hierarchical Representations [Nickel & Kiela, 2017]
- Word Translation without Parallel Data [Conneau et al., 2018]

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Word Embedding Limitations

- **Static representations (context independence)**: A word is always assigned a single vector representation regardless of its context
  - Words can have multiple meanings (polysemy)
  - Example: “bank” can mean a financial institution or the side of a river
- **Shallow representations**: Word embedding learning only focus on local context (a fixed window size of nearby words)
  - Cannot capture complex syntactic or long-range dependencies
  - Example: “The book that the president, who everyone admires, recommended is fascinating.” – distant subject (“book”) and adjective (“fascinating”)
- **Single-word representations**: Can only represent single words rather than larger linguistic units (phrases, sentences, paragraphs)
  - Many tasks require modeling relationships & compositionality between larger text chunks
  - Example: “They sell delicious hot dogs.” – “hot dogs” should be understood as an entire unit

Summary: Sparse vs. Dense Vectors

- Sparse vectors are derived based on frequencies/counts
  - High-dimensional – inefficiency in training & storage
  - Lots of zero dimensions – do not reflect semantics
- Dense vectors distribute information across multiple/all dimensions
  - Fewer dimensions; most dimensions are non-zero
  - More compact, robust, scalable, and efficient
  - Less interpretable

Summary: Word Embedding Learning

- Distributional hypothesis
  - Words that occur in similar contexts tend to have similar meanings
  - Infer semantic similarity based on context similarity
- Word embeddings
  - Construct a prediction task: use a center word's embedding to predict its contexts
  - Two words with similar embeddings will predict similar contexts => semantically similar
  - Word embedding is a form of self-supervised learning

Summary: Word2Vec

- Two variants: Skip-gram and CBOW
- Skip-gram: predict the words in a local context window surrounding the center word
- Employ negative sampling to improve training efficiency
- Use SGD to optimize vector representations
- Word embedding applications & evaluations
  - Word similarity
  - Word analogy
  - Use as input features to downstream tasks (e.g., text classification; NER)

Thank You!

Yu Meng
University of Virginia
yumeng5@virginia.edu