

Retrieval-Augmented Generation (RAG)

Slido: <https://app.sli.do/event/jsrxDoouM68J5dcrHyBMzJ>

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Oct 29, 2025

Overview of Course Contents

- Week 1: Logistics & Overview
- Week 2: N-gram Language Models
- Week 3: Word Senses, Semantics & Classic Word Representations
- Week 4: Word Embeddings
- Week 5: Sequence Modeling & Recurrent Neural Networks (RNNs)
- Week 6: Language Modeling with Transformers
- Week 8: Transformer and Pretraining
- Week 9: Large Language Models (LLMs) & In-context Learning
- **Week 10: Knowledge in LLMs and Retrieval-Augmented Generation (RAG)**
- Week 11: LLM Alignment
- Week 12: Reinforcement Learning for LLM Post-Training
- Week 13: LLM Agents + Course Summary
- Week 15 (after Thanksgiving): Project Presentations

Reminders

- Assignment 4 due next Monday 11/03 11:59pm
- First guest lecture next Monday 11/03 (Meet on Zoom; details to be shared later)

(Recap) Scaling Up Pretraining Data

The Pile: 22 sub-datasets (> 800GB), a common choice for pretraining corpus

Composition of the Pile by Category

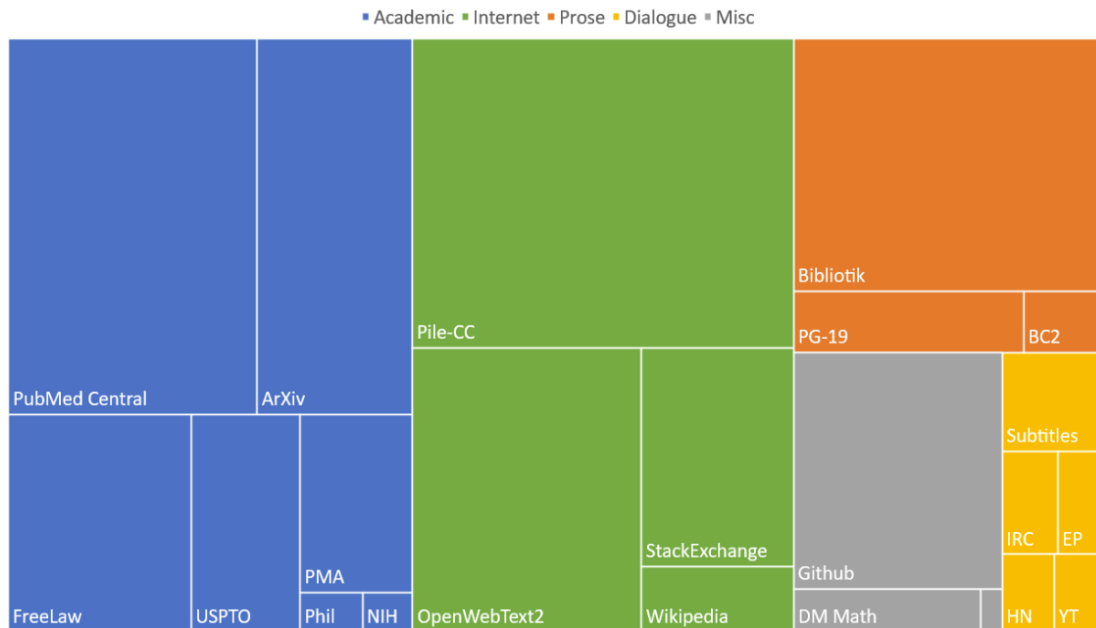
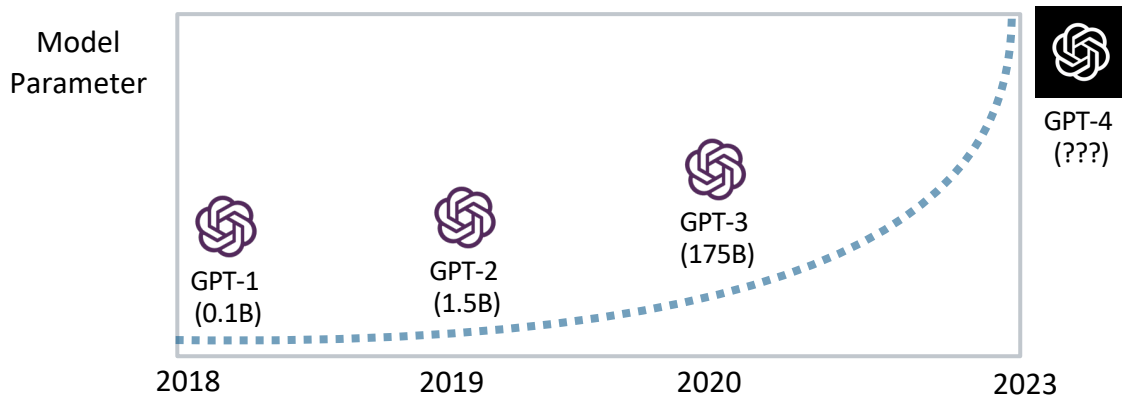


Figure source: <https://arxiv.org/pdf/2101.00027>

(Recap) Scaling Up Model Sizes

- GPT-1 (2018): 12 layers, 117M parameters, trained in ~1 week
- GPT-2 (2019): 48 layers, 1.5B parameters, trained in ~1 month
- GPT-3 (2020): 96 layers, 175B parameters, trained in several months



Papers: (GPT-1) https://cdn.openai.com/research-covers/language-unsupervised/language_understanding_paper.pdf

(GPT-2) https://d4mucfpksywv.cloudfront.net/better-language-models/language_models_are_unsupervised_multitask_learners.pdf

(GPT-3) <https://arxiv.org/pdf/2005.14165.pdf>



(Recap) Emergent Ability

- Larger models develop **emergent abilities**
 - Skills or capabilities that were not explicitly learned but arise as a result of model capacity
 - Larger models demonstrate surprising abilities in challenging tasks even when they were not explicitly trained for them
- Emergent capabilities typically become noticeable only when the model size reaches a certain threshold (cannot be predicted by small model's performance)

Emergent Abilities of Large Language Models

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Paper: <https://arxiv.org/pdf/2206.07682>

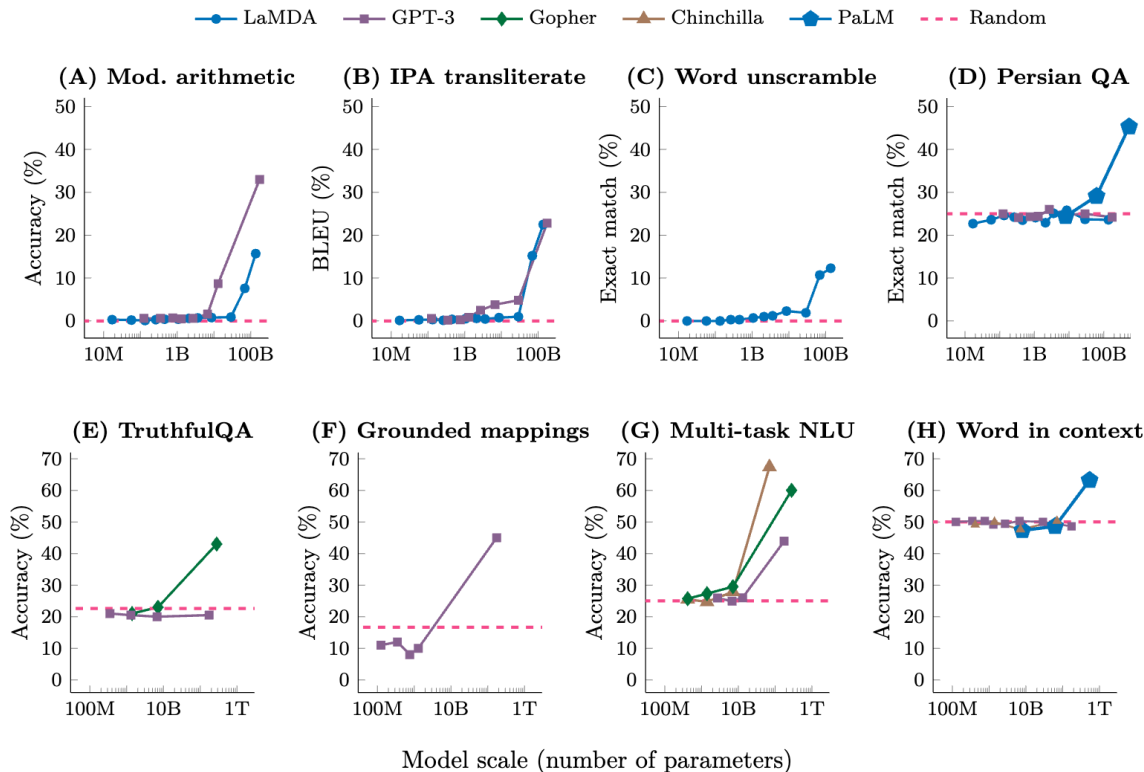
¹ Google Research ² Stanford University ³ UNC Chapel Hill ⁴ DeepMind



(Recap) Experiment Setting

- Consider the **few-shot in-context learning** paradigm
- Consider an ability to be **emergent** when a model has **random** performance until a certain scale, after which performance increases to **well-above random**
- Abilities to test
 - Arithmetic: addition, subtraction, multiplication
 - Transliteration
 - Recover a word from its scrambled letters
 - Persian question answering
 - Question answering (truthfully)
 - Grounded conceptual mappings
 - Multi-task understanding (math, history, law, ...)
 - Contextualized semantic understanding

(Recap) Performance vs. Model Scale



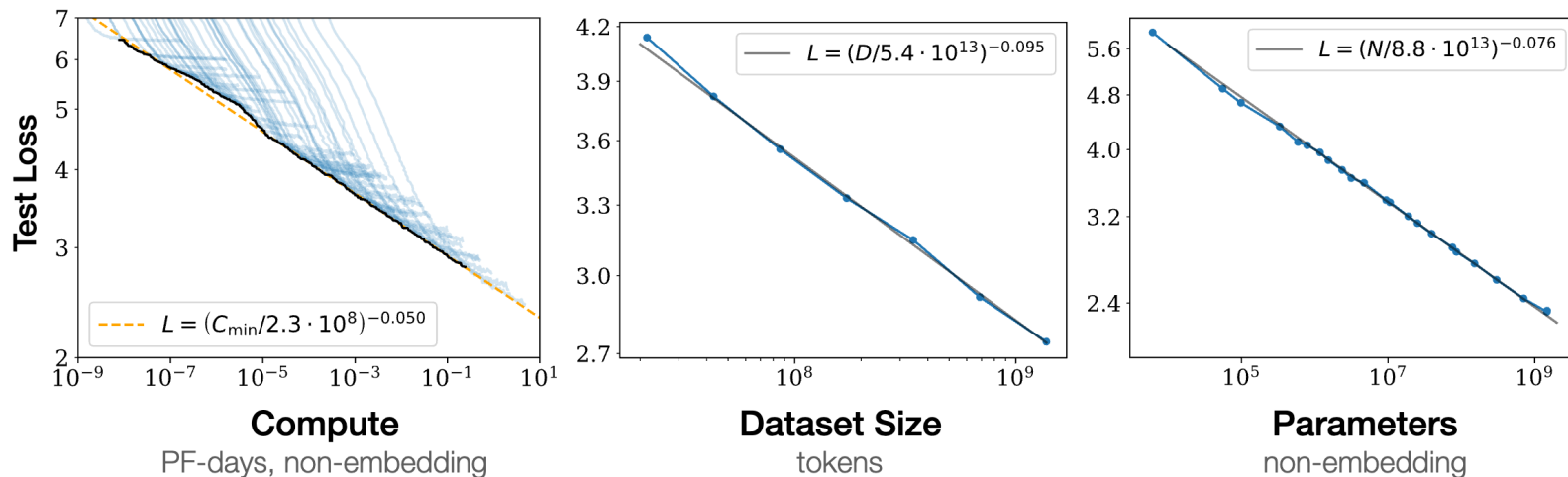
Models exhibit random performance until a certain scale, after which performance significantly increases

(Recap) Scaling Laws of LLMs

- (Pretrained) LLM performance is mainly determined by 3 factors
 - Model size: the number of parameters
 - Dataset size: the amount of training data
 - Compute: the amount of floating point operations (FLOPs) used for training
- Scaling up LLMs involves scaling up the 3 factors
 - Add more parameters (adding more layers or having more model dimensions or both)
 - Add more data
 - Train for more iterations
- **Scaling laws:** study the correlation between the cross-entropy language modeling loss and the above three factors
- How to optimally allocate a fixed compute budget?

(Recap) Scaling Laws of LLMs

Performance has a power-law relationship with each of the three scale factors (model size, dataset size, compute) when not bottlenecked by the other two

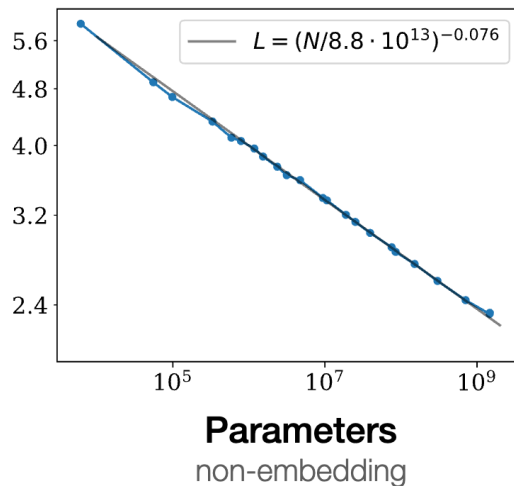


(Recap) Scaling Model Parameters

- Language model loss vs. models with a limited number of parameters (N)
 - Only count non-embedding parameters
 - Infinite compute: trained to convergence
 - Infinite dataset: trained with sufficiently large datasets
- Performance depends strongly on scale, weakly on model shape (depth vs. width)

$$\mathcal{L}(N) = \left(\frac{N_c}{N} \right)^{\alpha_N}, \quad \alpha_N \approx 0.076, \quad N_c \approx 8.8 \times 10^{13}$$


 Model parameters
 (non-embedding)

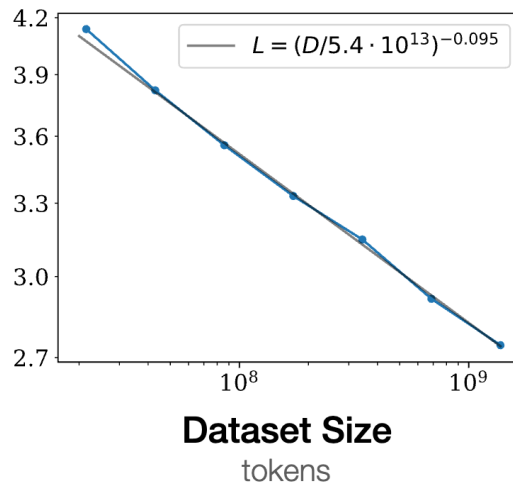


(Recap) Scaling Dataset Size

- Language model loss vs. a limited dataset size (D)
 - Infinite model size: sufficiently large model
 - With appropriate early stopping: avoid overfitting to the training data

$$\mathcal{L}(D) = \left(\frac{D_c}{D} \right)^{\alpha_D}, \quad \alpha_D \approx 0.095, \quad D_c \approx 5.4 \times 10^{13}$$


 Dataset size
 (# of tokens)

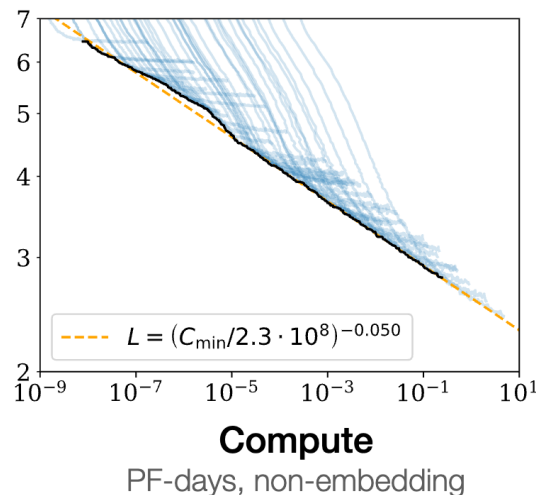


(Recap) Scaling Training Compute

- Language model loss vs. a limited amount of compute (C)
 - Infinite dataset size: sufficiently large training corpus
 - Optimal model size: can effectively learn the data and not excessively compute-consuming

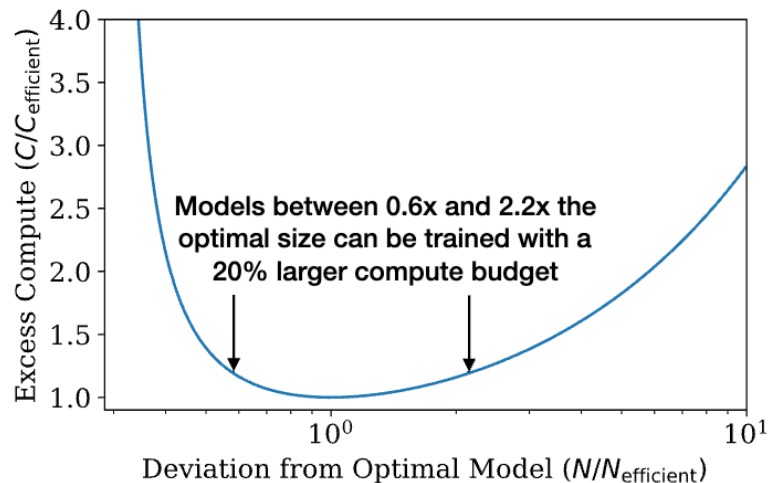
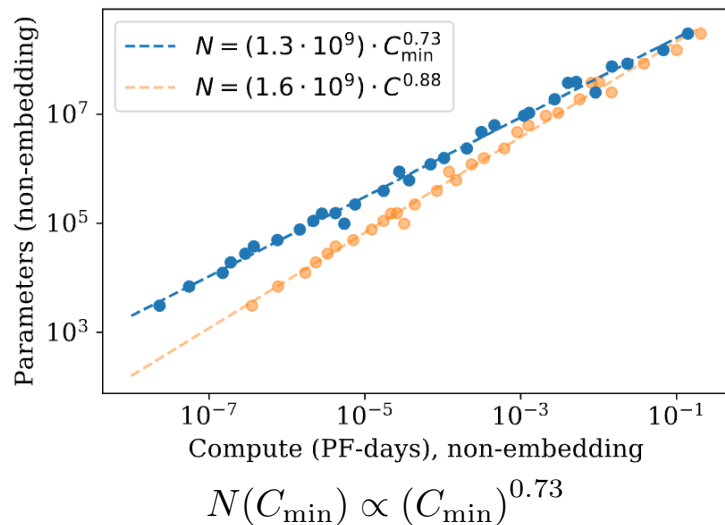
$$\mathcal{L}(C) = \left(\frac{C_c}{C} \right)^{\alpha_C}, \quad \alpha_C \approx 0.050, \quad C_c \approx 3.1 \times 10^8$$


 Compute
 (# Peta-FLOP days)



(Recap) Optimal Model Size

- Given a specific amount of training compute C , what's the optimal model size $N(C)$ that leads to minimal language modeling loss?
- $N(C)$ can be fit with a power-law wrt C
- Additional compute needs to be used when model size is suboptimal



(Recap) Introduction to Question Answering

- **Question Answering (QA):** build systems that can automatically answer questions posed by humans in natural language
- Categorization by application domain: closed-domain vs. open-domain QA
- **Closed-domain QA:** answer questions within a specific domain
 - Example: medical, legal, technical fields
 - Models are trained on specialized knowledge to be highly accurate within their domain
- **Open-domain QA:** answer questions from any domain
 - Typically rely on vast (external) knowledge sources like the web or large text corpora
 - Most LLM applications consider open-domain QA settings

(Recap) Introduction to Question Answering

- **Question Answering (QA):** build systems that can automatically answer questions posed by humans in natural language
- Categorization by modeling approach: extractive vs. abstractive QA
- **Extractive QA:** output a span of text extracted directly from a given context
 - A natural language understanding task (reading comprehension)
 - Example: context: “The human brain contains approximately 86 billion neurons” Q: “How many neurons are in the human brain?” A: “86 billion”
 - Can be done with encoder-only LMs (e.g., BERT)
- **Abstractive QA:** synthesize the answer in its own words (rephrasing/summarizing)
 - Example: context: “Albert Einstein published his theory of special relativity which introduced the famous equation $E=mc^2$, which relates energy (E) to mass (m) and the speed of light (c)” Q: “What did Einstein contribute to physics?” A: “Einstein made significant contributions to the theory of special relativity which established the relationship between energy and mass”
 - Need to use a generative LM (e.g., GPT)

(Recap) Introduction to Question Answering

- **Question Answering (QA):** build systems that can automatically answer questions posed by humans in natural language
- Categorization by access to external source: closed-book vs. open-book QA
- **Closed-book QA:** answer questions without access to any external information
 - Accuracy depends heavily on how well the training data covered the relevant information
 - Similar to a human answering a question from memory without looking anything up
- **Open-book QA:** can access external knowledge source to answer the questions
 - Typically using retrieval from reliable external sources that contain
 - Similar to a human answering a question by looking it up in a book or online resource

(Recap) Prompting LMs: Parametric Knowledge

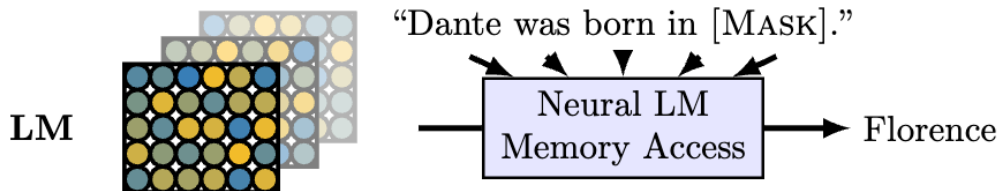
- LMs have learned from a lot of facts in their pretraining data
- LMs can be directly prompted to generate answers to factoid questions (Closed-book QA setting)
- Example:

$P(w|$ Q: Who wrote the book "The Origin of Species"? A:) prompt

- Since prompting LLMs only relies on the information stored within the parameters of the model itself, this kind of knowledge is called **parametric knowledge**

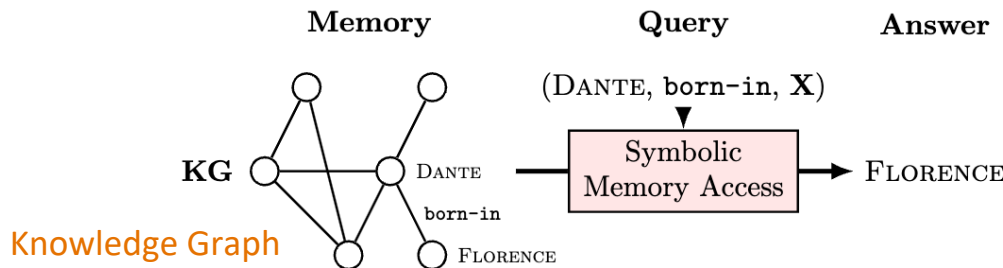
(Recap) Language Model as Knowledge Bases

- **Acquisition:** LM's knowledge is derived from the vast amount of pretraining data
- **Access:** information is accessed through natural language prompts
- **Update/maintenance:** re-training/fine-tuning the model with new data
- **Pros:**
 - Handle a wide range of natural language queries with contextual understanding
 - Generalize to unseen queries not seen during training
- **Cons:**
 - May produce incorrect/outdated information
 - Lack interpretability/transparency



(Recap) Real Knowledge Bases

- **Acquisition:** manually constructed by human annotators
- **Access:** information is accessed through queries in specific formats
- **Update/maintenance:** adding/modifying/deleting entries (incrementally) by humans
- **Pros:**
 - Precise & verifiable
- **Cons:**
 - Not able to handle natural language
 - Require massive human efforts to construct & maintain

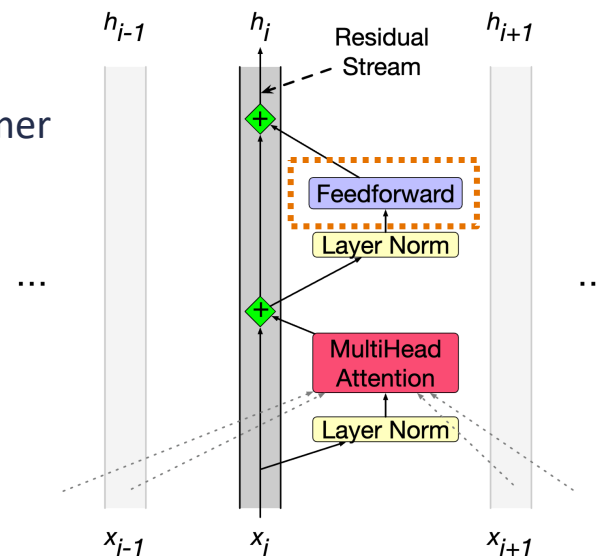


Feedforward Parameters in Transformer

- FFN in Transformer is a 2-layer network (one hidden layer, two weight matrices)

$$\text{FFN}(\mathbf{x}_i) = \text{ReLU}(\mathbf{x}_i \mathbf{W}_1) \mathbf{W}_2$$

- FFN constitutes $\sim 2/3$ of the total parameters of Transformer



Feedforward Parameters Are Neural Memories

Viewing FFN as key-value memories

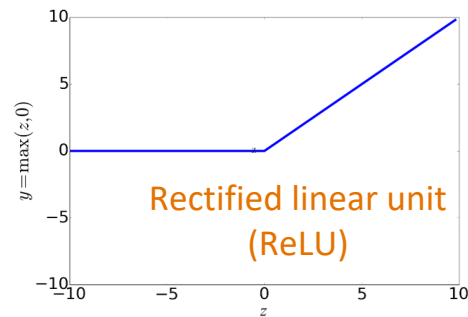
$$\text{FFN}(\mathbf{x}_i) = \text{ReLU}(\mathbf{x}_i \mathbf{W}_1) \mathbf{W}_2$$

↓

$\mathbf{x}_i \in \mathbb{R}^{d_1}$

$$\text{FFN}(\mathbf{x}_i) = \text{ReLU}(\mathbf{x}_i \mathbf{K}) \mathbf{V}$$

$\mathbf{K} \in \mathbb{R}^{d_1 \times d_2} \quad \mathbf{V} \in \mathbb{R}^{d_2 \times d_1}$



key vectors (column vectors in $\mathbf{K}$) act as **pattern detectors** over the input sequence

value vectors (row vectors in $\mathbf{V}$) represent **distributions over the output vocabulary**

$$\text{FFN}(\mathbf{x}_i) = \sum_{j=1}^{d_2} \text{ReLU}(\mathbf{x}_i \cdot \mathbf{k}_j) \mathbf{v}_j$$

weights of value vectors

Memory Keys Correspond to Input Patterns

Each individual key vector corresponds to a specific pattern over the input prefix

Key	Pattern	Example trigger prefixes
k_{449}^1	Ends with “substitutes” (shallow)	<i>At the meeting, Elton said that “for artistic reasons there could be no substitutes In German service, they were used as substitutes Two weeks later, he came off the substitutes</i>
k_{2546}^6	Military, ends with “base”/“bases” (shallow + semantic)	<i>On 1 April the SRSG authorised the SADF to leave their bases Aircraft from all four carriers attacked the Australian base Bombers flying missions to Rabaul and other Japanese bases</i>
k_{2997}^{10}	a “part of” relation (semantic)	<i>In June 2012 she was named as one of the team that competed He was also a part of the Indian delegation Toy Story is also among the top ten in the BFI list of the 50 films you should</i>
k_{2989}^{13}	Ends with a time range (semantic)	<i>Worldwide, most tornadoes occur in the late afternoon, between 3 pm and 7 Weekend tolls are in effect from 7:00 pm Friday until The building is open to the public seven days a week, from 11:00 am to</i>
k_{1935}^{16}	TV shows (semantic)	<i>Time shifting viewing added 57 percent to the episode’s The first season set that the episode was included in was as part of the From the original NBC daytime version , archived</i>



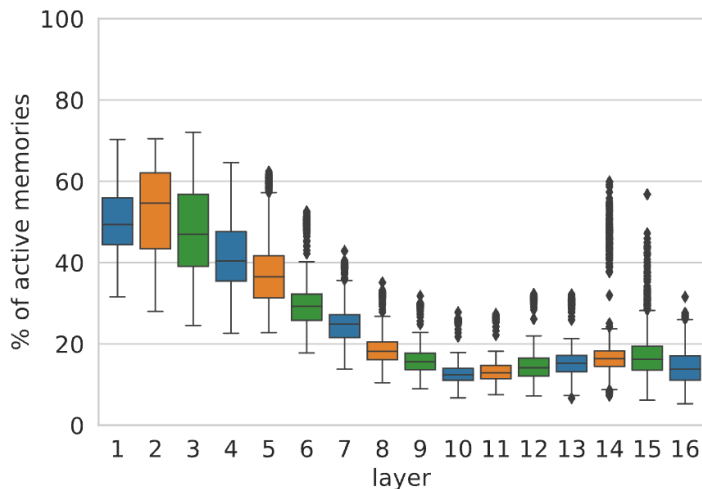
Memory Values Correspond to Output Tokens

Each value vector (roughly) matches a predicted token distribution

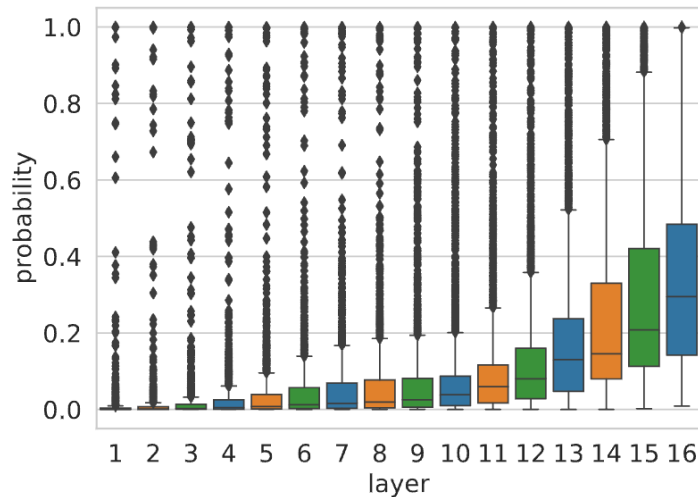
Value	Prediction	Trigger example
$\mathbf{v}_{222}^{15}$	<i>each</i>	<i>But when bees and wasps resemble each</i>
$\mathbf{v}_{752}^{16}$	<i>played</i>	<i>Her first role was in Vijay Lalwani's psychological thriller Karthik Calling Karthik, where Padukone was cast as the supportive girlfriend of a depressed man (played)</i>
$\mathbf{v}_{2601}^{13}$	<i>extratropical</i>	<i>Most of the winter precipitation is the result of synoptic scale, low pressure weather systems (large scale storms such as extratropical)</i>
$\mathbf{v}_{881}^{15}$	<i>part</i>	<i>Comet served only briefly with the fleet, owing in large part</i>
$\mathbf{v}_{2070}^{16}$	<i>line</i>	<i>Sailing from Lorient in October 1805 with one ship of the line</i>
$\mathbf{v}_{3186}^{12}$	<i>jail</i>	<i>On May 11, 2011, four days after scoring 6 touchdowns for the Slaughter, Grady was sentenced to twenty days in jail</i>

Memory Aggregation

- “Active” memories (memory vectors with non-zero coefficients) are typically sparse
- The residual connection sequentially refines token prediction from layer to layer



Fraction of active memory
units across layers



Output token probability is
gradually refined across layers

Further Reading on LLM Parametric Knowledge

- [How Much Knowledge Can You Pack Into the Parameters of a Language Model?](#) [Roberts et al., 2020]
- [Extracting Training Data from Large Language Models](#) [Carlini et al., 2021]
- [Locating and Editing Factual Associations in GPT](#) [Meng et al., 2022]

Agenda

- Hallucination
- Non-parametric Knowledge & Information Retrieval
- Sparse Retrieval (TF-IDF)
- Dense Retrieval
- Evaluation of Retrieval
- Retrieval-Augmented Generation (RAG) for LLMs

Hallucination

- **Hallucination:** LM generates information that is factually incorrect, misleading, or fabricated, even though it may sound plausible or convincing
- Why does hallucination happen?
 - Limited knowledge: LLMs are trained on finite datasets, which don't have access to all possible information; when asked about topics outside their training data, they may generate plausible-sounding but incorrect responses
 - Overgeneralization: LLMs may apply patterns they've learned from one context to another where they don't apply, leading to incorrect conclusions
 - Lack of common sense: While LLMs can process and generate human-like text, they often lack the ability to apply commonsense reasoning to their outputs
 - ...

Hallucination Examples

- **(Limited knowledge)** Q: “What were the main features of the iPhone 15 Pro Max?”
LLM (trained before 2023): “The iPhone 15 Pro Max features a revolutionary holographic display, quantum computing chip, and telepathic user interface.”
- **(Overgeneralization)** Q: “How do you form the past tense in Japanese?”
LLM: “In Japanese, you typically add '-ed' to the end of verbs to form the past tense, just like in English.” (incorrect)
- **(Lack of common sense)** Q: “How many tennis balls can fit in a typical smartphone?”
LLM: “Approximately 15-20 tennis balls can fit in a typical smartphone, depending on the model and screen size.”

what's your knowledge cutoff date?



My knowledge cutoff date is October 2023. This means I don't have information on events or developments that have occurred after that time. How can I assist you with your question?

Knowledge cutoff date of GPT4o

Concerns About Hallucination

BAKER DONELSON

[Home](#) > [Publications](#) > [The Perils of Legal Hallucinations and the Need for AI Training for Your In-House Legal Team](#)



Generative AI is rewriting how lawyers work at record speed, but is also quietly flooding Courts with phantom citations and invented case law along the way. In recent months, a number of judges across the country have sanctioned attorneys for submitting briefs laced with fictitious case law conjured by generative AI tools like ChatGPT. These so-called "hallucinations" aren't just embarrassing, they can constitute professional misconduct, jeopardize your company's interests, and damage the credibility of the legal profession in an already skeptical courtroom environment. In fact, since mid-2023, more than 120 cases of AI-driven legal "hallucinations" have been identified, with at least 58 occurring so far in 2025.

Figure source: <https://www.bakerdonelson.com/the-perils-of-legal-hallucinations-and-the-need-for-ai-training-for-your-in-house-legal-team>

Further Reading on Hallucination

- [LLM Lies: Hallucinations are not Bugs, but Features as Adversarial Examples](#) [Yao et al., 2023]
- [Towards Mitigating Hallucination in Large Language Models via Self-Reflection](#) [Ji et al., 2023]
- [Hallucination is Inevitable: An Innate Limitation of Large Language Models](#) [Xu et al., 2024]

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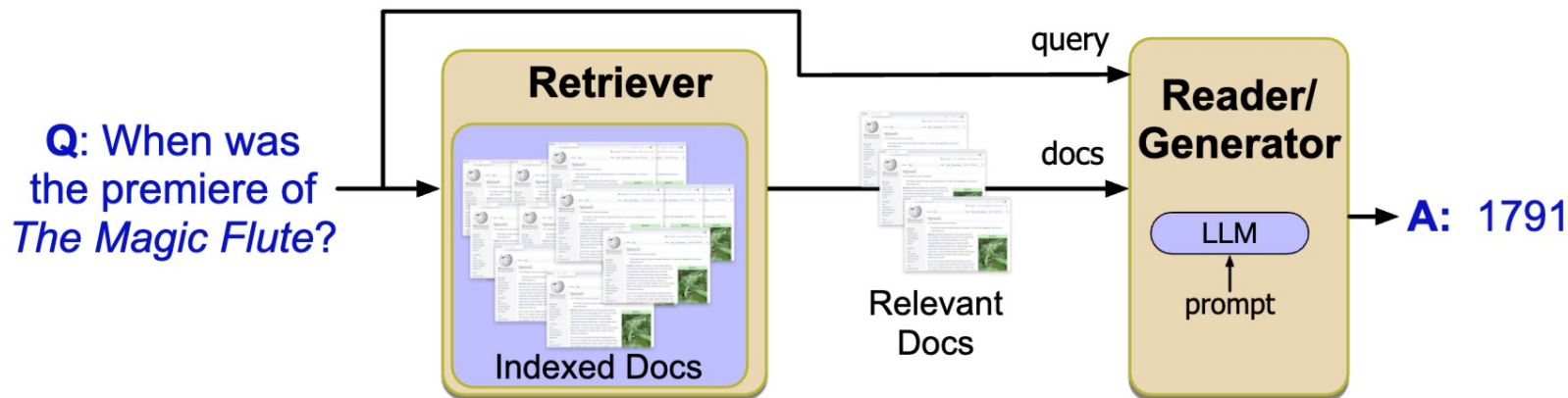
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Non-parametric Knowledge

- **Non-parametric knowledge:** (external) information not stored in the model's parameters but can be accessed or retrieved when needed
- Examples:
 - External knowledge bases/graphs
 - Pretraining corpora
 - User-provided documents/passages
- Non-parametric knowledge is typically used to **augment** parametric knowledge (typically via **retrieval**) for more accurate factoid question answering
- Benefits of **non-parametric knowledge**
 - Incorporate more information without increasing model size
 - Easier updates and modifications to the knowledge base
 - Improve model interpretability

Overview: Retrieval-Augmented Generation

- Use a **retriever** to obtain relevant documents to the query from an external text collection
- Use LLMs to generate answers given the documents and a prompt



Overview: Information Retrieval (IR)

- **Information retrieval (IR):** finding relevant information from a large collection of unstructured data (e.g., documents, web pages) in response to a user query
- **Query:** user-provided input (e.g., keywords or phrases), describing the information they are seeking
- **Documents/corpus:** the data collection that the system searches through
- **Ranking:** sort the search results by relevance based on specific metrics (e.g., keyword matching, semantic similarity)
- Web search engines (e.g., Google, Bing) are IR systems

Sparse vs. Dense Retrieval

- **Sparse** retrieval: based on traditional IR techniques where the representations of documents and queries are sparse (most vector values are zero)
 - Example: TF-IDF
 - Pros: simple and interpretable
 - Cons: lack semantic understanding
- **Dense** retrieval: encode documents and queries into dense vectors (embeddings) using deep neural networks
 - Example: BERT-based encoding methods
 - Pros: semantic & contextualized understanding
 - Cons: computationally more expensive and less interpretable

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TF-IDF Weighting

- Introduced in week 3's lectures $\text{TF-IDF}(w, d) = \text{TF}(w, d) \times \text{IDF}(w)$
- Main idea: represent a document with frequent & distinctive words

TF-IDF weighted

	As You Like It	Twelfth Night	Julius Caesar	Henry V
battle	0.246	0	0.454	0.520
good	0	0	0	0
fool	0.030	0.033	0.0012	0.0019
wit	0.085	0.081	0.048	0.054

$$\cos(\mathbf{v}_{d_2}, \mathbf{v}_{d_3}) = 0.10 \quad \cos(\mathbf{v}_{d_3}, \mathbf{v}_{d_4}) = 0.99$$

Raw counts

	As You Like It	Twelfth Night	Julius Caesar	Henry V
battle	1	0	7	13
good	114	80	62	89
fool	36	58	1	4
wit	20	15	2	3

$$\cos(\mathbf{v}_{d_2}, \mathbf{v}_{d_3}) = 0.81 \quad \cos(\mathbf{v}_{d_3}, \mathbf{v}_{d_4}) = 0.99$$

Term Frequency (TF)

- A word appearing 100 times in a document doesn't make it 100 times more likely to be relevant to the meaning of the document
- Instead of using the raw counts, we squash the counts with log scale

$$\text{TF}(w, d) = \begin{cases} 1 + \log_{10} \text{count}(w, d) & \text{count}(w, d) > 0 \\ 0 & \text{otherwise} \end{cases}$$

Inverse Document Frequency (IDF)

- We want to emphasize discriminative words (with low DF)
- Inverse document frequency (IDF): total number of documents (N) divided by DF, in log scale

$$\text{IDF}(w) = \log_{10} \left(\frac{N}{\text{DF}(w)} \right)$$

Word	df	idf
Romeo	1	1.57
salad	2	1.27
Falstaff	4	0.967
forest	12	0.489
battle	21	0.246
wit	34	0.037
fool	36	0.012
good	37	0
sweet	37	0

DF & IDF statistics in the
Shakespeare corpus
(37 documents)

TF-IDF for Sparse Retrieval

- Score document-query semantic similarity by cosine similarity

$$\cos(\mathbf{q}, \mathbf{d}) = \frac{\mathbf{q} \cdot \mathbf{d}}{|\mathbf{q}| |\mathbf{d}|}$$

- Both document and query vectors use TF-IDF weighting
- Can also adopt other weighting schemes (e.g., BM25)

Example: TF-IDF for Sparse Retrieval

- Example query and mini-corpus:

Query: sweet love

Doc 1: Sweet sweet nurse! Love?

Doc 2: Sweet sorrow

Doc 3: How sweet is love?

Doc 4: Nurse!

- Query & document vectors:

word	Query					
	cnt	tf	df	idf	tf-idf	n'lized = tf-idf/ q
sweet	1	1	3	0.125	0.125	0.383
nurse	0	0	2	0.301	0	0
love	1	1	2	0.301	0.301	0.924
how	0	0	1	0.602	0	0
sorrow	0	0	1	0.602	0	0
is	0	0	1	0.602	0	0

word	Document 1			
	cnt	tf	tf-idf	n'lized
sweet	2	1.301	0.163	0.357
nurse	1	1.000	0.301	0.661
love	1	1.000	0.301	0.661
how	0	0	0	0
sorrow	0	0	0	0
is	0	0	0	0

	Document 2			
	cnt	tf	tf-idf	n'lized
1	1	1.000	0.125	0.203
0	0	0	0	0
0	0	0	0	0
0	0	0	0	0
1	1	1.000	0.602	0.979
0	0	0	0	0

$$\cos(\mathbf{q}, \mathbf{d}_1) = 0.747$$

$$\cos(\mathbf{q}, \mathbf{d}_2) = 0.078$$

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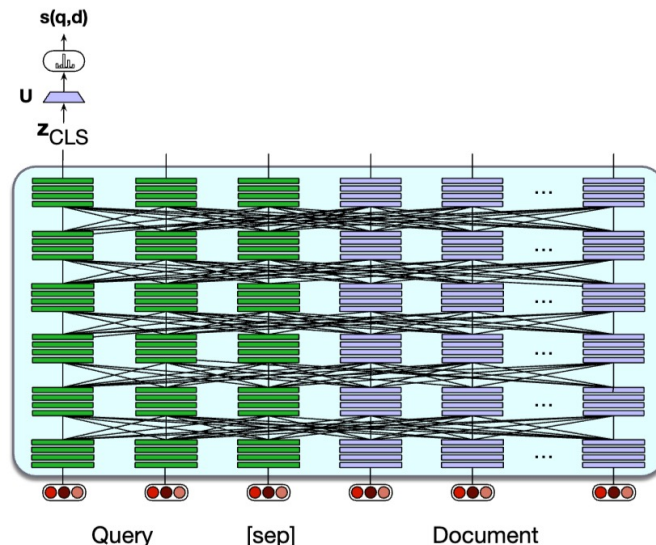
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Dense Retrieval

- Motivation: sparse retrieval (e.g., TF-IDF) relies on the exact overlap of words between the query and document without considering semantic similarity
- Solution: use a language model to obtain (dense) distributed representations of query and document
- The retriever language model is typically a small text encoder model (e.g., BERT)
 - Retrieval is a natural language understanding task
 - Encoder-only models are more efficient than LLMs for this purpose
- Both query and document representations are computed by text encoders

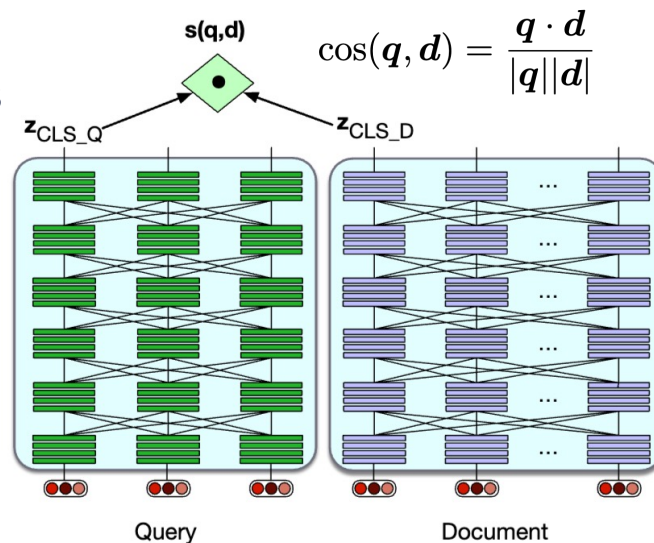
Dense Retrieval: Cross-encoder

- Process query-document pairs together
- Relevance score produced directly by the model output
- (+) Capture intricate interactions between the query and the document
- (-) Not scalable to large retrieval corpus
- Good for small document sets



Dense Retrieval: Bi-encoder

- Independently encode the query and the document using two separate (but often identical) encoder models
- Use cosine similarity between the query and document vectors as relevance score
- (+) Document vectors can be precomputed
- (-) Cannot capture query-document interactions
- Common choice for large-scale retrieval



Agenda

- Hallucination
- Non-parametric Knowledge & Information Retrieval
- Sparse Retrieval (TF-IDF)
- Dense Retrieval
- Evaluation of Retrieval
- Retrieval-Augmented Generation (RAG) for LLMs

Evaluation of IR Systems

- Assume that each document returned by the IR system is either **relevant** to our purposes or **not relevant**
- Given a query, assume the system returns a set of ranked documents T
 - A subset R of these are relevant (The remaining $N = T - R$ is irrelevant)
 - There are U documents in the entire retrieval collection that are relevant to this query
- **Precision:** the fraction of the returned documents that are relevant

$$\text{Precision} = \frac{|R|}{|T|}$$

- **Recall:** the fraction of all relevant documents that are returned

$$\text{Recall} = \frac{|R|}{|U|}$$

Precision & Recall @ k

- We hope to build a retrieval system that ranks the relevant documents higher
- Use precision & recall @ k (among the top- k items in the ranked list) to reflect this

Rank	Judgment	Precision _{Rank}	Recall _{Rank}
1	R	1.0	.11
2	N	.50	.11
3	R	.66	.22
4	N	.50	.22
5	R	.60	.33
6	R	.66	.44
7	N	.57	.44
8	R	.63	.55
9	N	.55	.55
10	N	.50	.55

Assume there are 9 total relevant documents in the retrieval corpus

Average Precision

Average precision (AP): mean of the precision values at the points in the ranked list where a relevant document is retrieved

Indicator function of whether
the document is relevant

$$AP = \frac{1}{|R|} \sum_{k=1}^{|T|} (\text{Precision@}k \times \mathbb{1}(d_k \text{ is relevant}))$$

Rank	Judgment	Precision _{Rank}	Recall _{Rank}
1	R	1.0	.11
2	N	.50	.11
3	R	.66	.22
4	N	.50	.22
5	R	.60	.33
6	R	.66	.44
7	N	.57	.44
8	R	.63	.55
9	N	.55	.55
10	N	.50	.55

Agenda

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RAG for LLMs

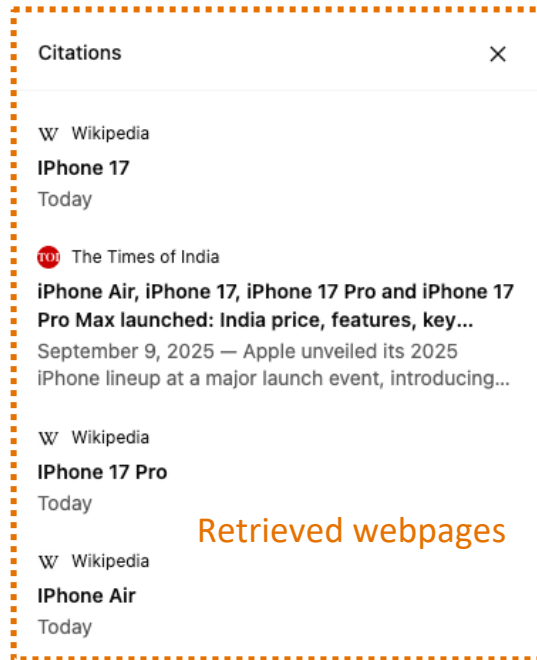
RAG is a common technique to enhance factuality of LLM generation & mitigate hallucination

Time-sensitive query triggers
RAG for ChatGPT

What is the latest iPhone model?

Searching for latest iPhone model Apple

Retriever performs web search



The screenshot shows the 'Citations' panel in ChatGPT, which lists sources used to generate the response. The panel is titled 'Citations' with a close button (X) in the top right corner. It contains three entries, each with a source icon, the source name, a bolded title, and a date. The first entry is from Wikipedia, titled 'iPhone 17', dated 'Today'. The second entry is from 'The Times of India', titled 'iPhone Air, iPhone 17, iPhone 17 Pro and iPhone 17 Pro Max launched: India price, features, key...', dated 'September 9, 2025'. The third entry is from Wikipedia, titled 'iPhone 17 Pro', dated 'Today'. The text 'Retrieved webpages' is overlaid in orange on the right side of the panel.

Citations

W Wikipedia
iPhone 17
Today

TOI The Times of India
iPhone Air, iPhone 17, iPhone 17 Pro and iPhone 17 Pro Max launched: India price, features, key...
September 9, 2025 — Apple unveiled its 2025 iPhone lineup at a major launch event, introducing...

W Wikipedia
iPhone 17 Pro
Today

W Wikipedia
iPhone Air
Today

Retrieved webpages

RAG vs. Direct Prompting

- Prompting relies on LM's parametric knowledge to directly answer the question:

$P(w|Q)$ Q: Who wrote the book "The Origin of Species"? A: prompt

- RAG prepends the set of retrieved passages to the question

Schematic of a RAG Prompt

retrieved passage 1

retrieved passage 2

...

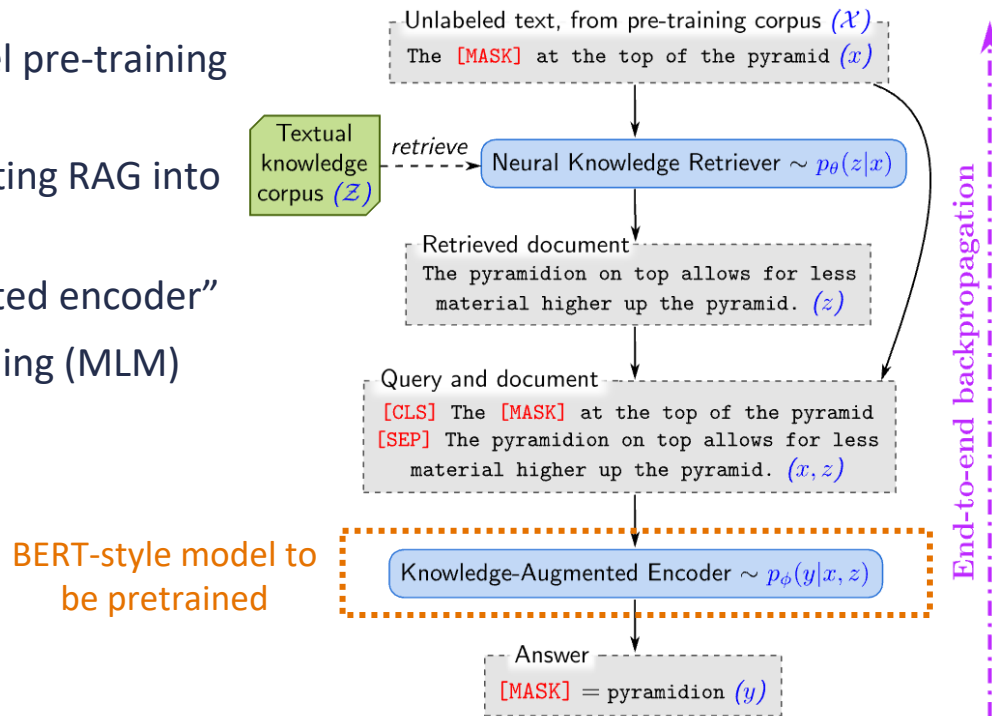
retrieved passage n

Returned by the retriever

Based on these texts, answer this question: Q: Who wrote the book "The Origin of Species"? A:

RAG in Pretraining

- Retrieval-Augmented Language Model pre-training (REALM)
- The first paper that studies incorporating RAG into encoder pretraining (BERT style)
- Main model is a “knowledge-augmented encoder”
- Pretrain with masked language modeling (MLM) loss conditioned on retrieved content



BERT-style model to be pretrained



RAG for Text Generation

- The first paper that studies RAG for text generation
- Both the retriever (an encoder-only LM) and the generator (an LLM) are trained

Retrieval-Augmented Generation for Knowledge-Intensive NLP Tasks

Patrick Lewis^{†‡}, Ethan Perez^{*},

Aleksandra Piktus[†], Fabio Petroni[†], Vladimir Karpukhin[†], Naman Goyal[†], Heinrich Küttler[†],

Mike Lewis[†], Wen-tau Yih[†], Tim Rocktäschel^{†‡}, Sebastian Riedel^{†‡}, Douwe Kiela[†]

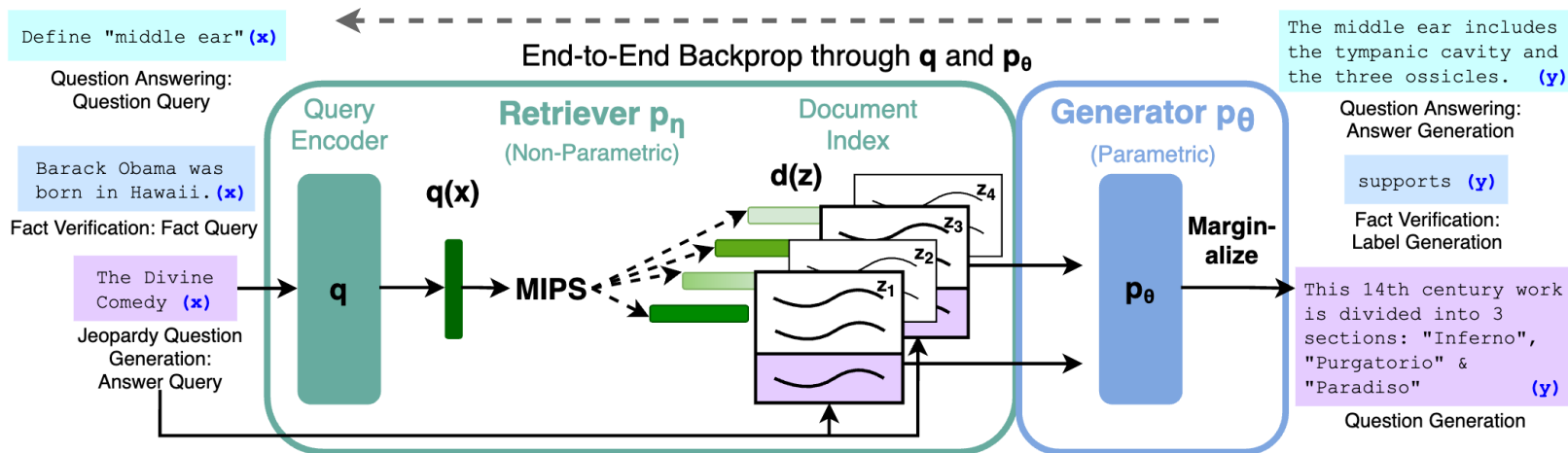
A Latent Variable Model

The retrieved documents are treated as latent variables (z) for generation

$$p(\mathbf{y}|\mathbf{x}) = \sum_{\mathbf{z} \in \mathcal{D}} p(\mathbf{z}|\mathbf{x})p(\mathbf{y}|\mathbf{x}, \mathbf{z})$$

Retrieve document (z)
based on query (x)

Generate answer (y) based on
retrieved docs (z) and query (x)





RAG-Sequence Model

- Use the same retrieved document to generate the complete sequence
- Treat the retrieved document as a single latent variable
- Marginalize to get the generation probability $p(\mathbf{y}|\mathbf{x})$ via a top-K approximation

$$p_{\text{RAG-sequence}}(\mathbf{y}|\mathbf{x}) \approx \sum_{\mathbf{z} \in \text{top-K}(p(\cdot|\mathbf{x}))} p_{\eta}(\mathbf{z}|\mathbf{x}) p_{\theta}(\mathbf{y}|\mathbf{x}, \mathbf{z}) = \sum_{\mathbf{z} \in \text{top-K}(p(\cdot|\mathbf{x}))} p_{\eta}(\mathbf{z}|\mathbf{x}) \prod_{i=1}^N p_{\theta}(y_i|\mathbf{x}, \mathbf{z}, \mathbf{y}_{<i})$$

 Top-K approximation
(only consider the top-K retrieved docs)

 The same retrieved doc (z) is used to generate all tokens in the sequence



RAG-Token Model

- Can use different retrieved documents to generate different tokens in a sequence
- Marginalization is performed for each generated token (rather than at sequence level)

$$p_{\text{RAG-token}}(\mathbf{y}|\mathbf{x}) = \prod_{i=1}^N p_{\theta}(y_i|\mathbf{x}, \mathbf{y}_{<i}) \approx \prod_{i=1}^N \sum_{\mathbf{z} \in \text{top-K}(p(\cdot|\mathbf{x}, \mathbf{y}_{<i}))} p_{\eta}(\mathbf{z}|\mathbf{x}, \mathbf{y}_{<i}) p_{\theta}(y_i|\mathbf{x}, \mathbf{z}, \mathbf{y}_{<i})$$



Different retrieved doc ($\mathbf{z}$) can be used to generate different tokens in the sequence

RAG-Sequence & RAG-Token Results

Evaluation results on open-domain QA tasks:

- Natural Questions (NQ)
- TriviaQA (TQA)
- WebQuestions (WQ)
- CuratedTrec (CT)

Model		NQ	TQA	WQ	CT
Closed	T5-11B [52]	34.5	- /50.1	37.4	-
Book	T5-11B+SSM[52]	36.6	- /60.5	44.7	-
Open	REALM [20]	40.4	- / -	40.7	46.8
Book	DPR [26]	41.5	57.9 / -	41.1	50.6
RAG-Token		44.1	55.2/66.1	45.5	50.0
RAG-Seq.		44.5	56.8/ 68.0	45.2	52.2



Further Reading on RAG

- [Generalization through Memorization: Nearest Neighbor Language Models](#) [Khandelwal et al., 2019]
- [Active Retrieval Augmented Generation](#) [Jiang et al., 2023]
- [Self-RAG: Learning to Retrieve, Generate, and Critique through Self-Reflection](#) [Asai et al., 2023]
- [InstructRAG: Instructing Retrieval-Augmented Generation via Self-Synthesized Rationales](#) [Wei et al., 2024]



Thank You!

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